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**A MODIFIED ANDERSON DARLING
GOODNESS-OF-FIT TEST
FOR THE GAMMA DISTRIBUTION WITH
UNKNOWN SCALE AND LOCATION
PARAMETERS**

THESIS

Tamer ÖZMEN

First Lieutenant, TUAF

AFIT/GOR/ENS/93M-24

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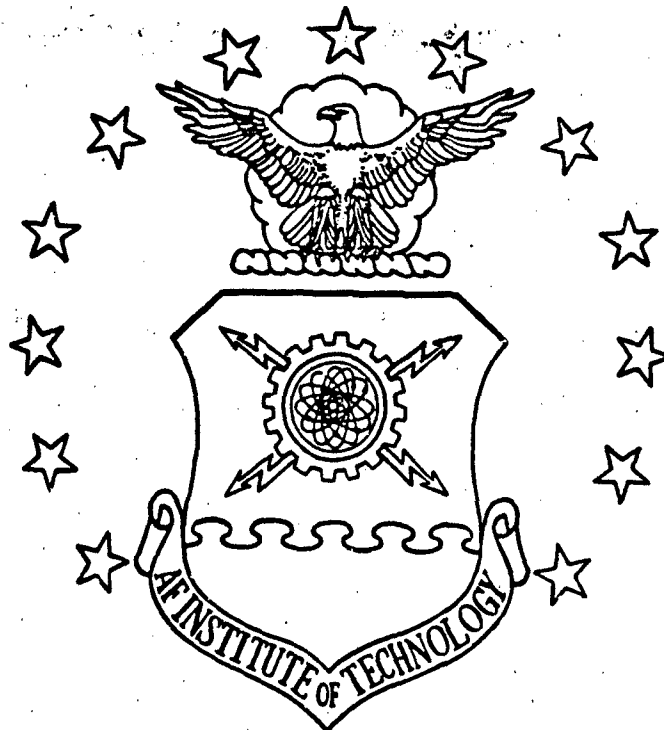
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**A MODIFIED ANDERSON DARLING
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THESIS

**Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University**

**In Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Operations Research**

**Tamer ÖZMEN, B.S.
First Lieutenant, TUAF**

March, 1993

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Preface

This thesis is an extension of the works in the field of parameter estimation of the three parameter Gamma distribution. It uses and compares a new technique of estimation by using Anderson Darling test statistics keeping the shape parameter known but scale and location unknown. And a power study is proposed to see the effectiveness of the new modified test.

My appreciations go first to Dr. Albert H. Moore as my thesis advisor who made valuable suggestions for improvements in my thesis. He really helped and led me during this thesis effort, with his great experiences on statistics and especially on Goodness-of-Fit techniques. I am indepted to him. This thesis could not be done without his help, support, understanding, patience and encouragement. I also express my appreciations to Dr. Joseph P. Cain for being my reader and his comments and assistance.

Thanks are given to all the instructors and the personel of Departments of Operational Sciences and Mathematical Sciences. They taught me; they helped me; and they were so patient. Also, I would like to say thanks to all my classmates and especially to my best friend E. Yücel because of being with me all the time and encouraging me. And I will never forget my parents Mehmet and Sengül's support from thousands of miles away.

I will be forever grateful to my country, to my people because of giving me this opportunity. I hope, I will have chances to pay it back.

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Tamer ÖZMEN

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Abstract

A new modified Anderson-Darling (A-D) goodness-of-fit test is introduced for the three-parameter Gamma distribution when the location parameter is found by minimum distance estimation and scale parameter by maximum likelihood estimation. Monte Carlo simulation studies were performed to calculate the critical values for A-D test when A-D statistic is minimized. These critical values are then used for testing whether a set of observations follows a Gamma distribution when the scale and location parameters are unspecified and are estimated from the sample. In this research, Monte Carlo simulation used 5000 repetitions for sample sizes of 5 through 40 with step 5. Gamma shape parameter is taken 0.5 through 4 with step 0.5. Functional relationship between the critical values of A-D is also examined for each shape parameter by the variables, sample size (n) and significance level (α).

The power study is performed for sample sizes 5 through 30 with the hypothesized Gamma shape parameter 0.5, 1.0, 1.5, 2.0, 3.0 against alternate distributions. The significance levels 0.01, 0.05, 0.10, 0.15, 0.20 are used during the entire work. Also, for shape parameters 1.5 and 4.0 (with two significance levels and three sample sizes) some power comparisons are made with the test which uses MLEs for location and scale. In most cases, the modified test looks better.

A MODIFIED ANDERSON DARLING GOODNESS-OF-FIT TEST FOR THE GAMMA DISTRIBUTION WITH UNKNOWN SCALE AND LOCATION PARAMETERS

I. INTRODUCTION

1.1 Background

Nations face many challenges in the national security environment. These challenges cause development of the weapon and the support systems. New technologies increase the complexity of the new systems and these complexities can cause prices to climb higher and higher. Because of the nation's limited defense budget the reliability of these systems becomes more important. "The reliability of a system is the probability that, when operating under stated environmental conditions, the system will perform its intended function adequately for a specified interval of time [30:1]." Reliability analysis uses some statistical techniques to find out the failure rate of the systems or the components of the systems during their operational life. Estimations about the failure rate are based on the model function. Well-estimated parameters of the model function can provide more accurate results on the reliability analysis. Probabilistic or statistical modeling and associated analyses have a great importance in making decision in diverse fields. Choosing an inappropriate model can make the analyses meaningless or useless.

"Improve constantly and forever every activity."

"The Japanese call this *kaizen*, meaning continuous searching for incremental improvement. And they believe, it can be achieved by integrating the research and development effort with the actual production facility and devoting a lot of time and money to "getting the process right" and then continually making it better. The key factor is the pervasive use of statistical methods [29:4]." The need for statistics flows naturally from the recognition that data collection and analysis is necessary to the solution of quality problems. One of the most important factors that contributed to the success of management is the utilization of modern statistical methods for quality control and reliability. Good decisions are based on facts, not opinions and emotions [25:v]. The British physicist Lord Kelvin put it this way:

"When you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of the meager and unsatisfactory kind [29:5]."

Data collection is one of the most important phases of the statistical modeling. After the collection of the data, statisticians try to estimate the probability distribution which most closely fits the data. Next, goodness-of-fit tests put forward how well the hypothesized distribution corresponds to the sample data. Cohen and Whitten [12] points out the the important role of the skewed distributions in the analysis of sample data originating from life span, reaction time, reliability, survivor and related studies. They are also useful as models for life tests and models for distributions of some items of interest in economic and financial studies. The **Gamma** is a distribution which is positively skewed and along with the Weibull, lognormal and inverse Gaussian distributions. In some details it resembles the Weibull. James calls attention to this "The Gamma distribution has the flexibility to describe a mixture of chance and wearout failures [28:1]. It has been widely used to model the time-to-failure distribution of electrical, mechanical, and electro-mechanical systems

[6:243].” Kapur and Lamberson [30], Law and Kelton [32], Scheaffer and McClave [43], Hogg and Ledolter [25], and Ireson [27] provide some numerical examples and illustrate the appropriate use of the distribution on the areas given above. This research addresses the problem of estimating the location and scale parameter of a three-parameter Gamma distribution when the shape parameter is fixed as a constant. It consists of the *distance estimation* technique for the location parameter and *maximum likelihood* for the scale parameter.

After having the data sample set from the specified model, statistical theory approaches to the estimation by the way of trying to determine the properties of the parameters of the specified model. The specified model is a density function of a known family of distributions. Several estimation techniques can be found to estimate the parameters of a statistical distribution from observed data. Some of the well-known estimating methods are the method of moments, least squares, Bayes, chi-square, maximum likelihood and minimum distance.

In summary, the minimum distance (MD) estimation method minimizes the distance between the cumulative distribution function (CDF) values of the sample data and the empirical distribution function (EDF). The *distance* between the CDF and EDF is measured by a goodness-of-fit statistic. The method adjusts the parameters until the distance is minimized. The final result is the estimate of the parameters. The likelihood function tells us how likely the observed sample is as a function of the possible parameter values. Maximizing the likelihood for data gives the parameter values for which the observed sample is most likely to have been generated – that is, the parameter values that “agree most closely” with the observed data [16:247-248]. The value of a given parameter that maximizes the likelihood function is called a maximum likelihood estimate (MLE) of that parameter.

1.2 Problem Statement

There has been many studies completed in the areas of parameter estimation of the three parameter Gamma distribution. minimum distance estimation and maximum likelihood estimation techniques, however in this thesis I have used both techniques respectively for location parameter and scale parameter keeping the shape parameter constant. The objective of the research was to derive critical values for a new test statistic, to examine the power of the new test and to put forward a comparison with the previous study in which MLE had been used in. The Anderson-Darling (A-D) test statistic has been chosen because the current knowledge -according to the previous studies- encourages to use this statistic. It is satisfactory with distributions having tails similar to the Gamma distribution.

1.3 Assumptions

In this thesis effort, we assumed that the shape parameter of the three-parameter Gamma distribution as known and location and scale parameters are not known. The location parameter determines the guaranteed life, the scale parameter determines the relative scale of the distribution and they both will be estimated. Another assumption, x_1, \dots, x_n is a complete random sample from an unknown distribution.

1.4 Scope

The goal of the research was to derive a new Anderson-Darling (A-D) test statistic with accompanying critical values and to provide a useful tool to reliability test engineers. After the critical values are derived a power study will be performed.

1.5 Methodology

The methodology describes the steps that will be taken to achieve the objective. In this effort, the first step was generation of N random Gamma deviates. I used sample size 5 through 40 with a step of 5 (5,5,40) for the critical values and 5 through 30 with a step of 5 (5,5,30) for the power study. Next steps include calculation of MLE's as the initial estimates of the unknown parameters, location and scale and recalculation of location by MD and scale by MLE. After calculation of goodness-of-fit statistics, critical values have been found.

During the power study, N random deviates have been generated from alternate distributions. The critical values obtained in the first section of the study have been used in the study for every shape parameter. Rejection numbers have been taken and the rejection percentages have been used as the power. In both of the studies 5000 repetition was applied to the steps. The detailed information and the tables for critical values and the power studies are proposed in the following chapters.

1.6 Support Materials and Equipment

This research requires some computer routines to perform the steps given in the methodology. AFIT computer resources (SUN SPARC STATIONS) have been used during the studies. Two Fortran codes for the critical values and power study were prepared and given on appendices A and B respectively. Verification of the codes are ensured by Mathcad (A mathematical software available on the school systems). All different steps gave the same results and numbers.

II. LITERATURE REVIEW

2.1 Introduction

One of the fundamental tasks of engineering and science, and indeed of mankind in general, is the extraction of information from data. Parameter estimation is a discipline that provides tools for the efficient use of data in the estimation of constants appearing in mathematical models and for aiding in modeling of phenomena [4:1]. Unfortunately there is no "regular" method to obtain efficient estimators of parameters of density based on a sample of a fixed size [48:73]. Difficulties arise in estimating parameters in experiments for which the parameters themselves are not directly measureable.

The derivations and application of modern estimators and estimation algorithms are buried in the technical literature on communication theory, statistics, control theory and others. Estimation theory originated with in the broad area of statistics and gradually found its way through many disciplines of science and engineering [38].

Many studies have been accomplished in the areas of parameter estimation, goodness-of-fit techniques and skewed distributions. A literature search has been completed on the random number generation, maximum likelihood estimation, minimum distance method and goodness-of-fit tests. The following paragraphs will review several parts of the research and some others will be included in the discussion of the methodology.

2.2 Discussion

2.2.1 Maximum Likelihood Estimation. The method of maximum likelihood has been recommended by many statisticians, at least when the sample size is large, since the resulting estimators have certain desirable efficiency properties. The likelihood function tells us how likely the observed sample is as a function of the possible parameter values. Maximizing the likelihood gives the parameter values for which the observed sample is most likely to have been generated - that is, the parameter values that "agree most closely" with the observed data [16:247-248]. The value of a given parameter that maximizes the likelihood function is called a maximum likelihood estimate of that parameter.

Let x_1, \dots, x_n be a set of n independent random samples drawn from a given population. The population is characterized by a probability density function $f_n(x; \theta) = f_n(x_1, \dots, x_n; \theta)$. θ is a parameter of the population distribution. The likelihood function L is defined in this case by the relation

$$L(x_1, \dots, x_n; \theta) = f_n(x_1, \dots, x_n; \theta) \quad (2.1)$$

$$= f(x_1; \theta) f(x_2; \theta) \dots f(x_n; \theta) \quad (2.2)$$

$$= \prod_{i=1}^n f(x_i; \theta) \quad (2.3)$$

In other words, when the sample values are given as well as the function form of the population probability density function, the likelihood function can be regarded as a function of the distribution parameter θ . A similar relation holds in which the density functions are replaced by the probability distribution functions. The method of maximum likelihood is one of selecting an estimate $\hat{\theta}$ for θ which will maximize the likelihood function L . Since $\ln L$ is a monotonic function and attains its maximum when L is a maximum, and the likelihood function given above is usually solved for

the estimate $\hat{\theta}$ by considering the simpler expression

$$\frac{\partial}{\partial \theta} \ln L = 0 = \frac{\partial}{\partial \theta} \sum_{i=1}^n \ln f(x_i; \theta), \quad (2.4)$$

rather than the usually cumbersome form of

$$\frac{\partial}{\partial \theta} L = 0 = \frac{\partial}{\partial \theta} \prod_{i=1}^n f(x_i; \theta) \quad (2.5)$$

Any solution $\hat{\theta}$ for θ which satisfies the equation above and is identically a constant is called an MLE (*Maximum Likelihood Estimate*) of θ . Above equation is called the *likelihood equation* [15:134].

"The method of maximum likelihood was first introduced by R.A. Fisher, a geneticist and statistician, in the 1920's [16:247]. Maximum likelihood is the most common and accepted method of parameter estimation. This method selects values as estimates that maximize the likelihood of the observed sample, where the likelihood function is the joint density function [35:419]." Harter and Moore presented an iterative algorithm to compute the maximum likelihood estimates (MLEs) for the Gamma and Weibull distributions [21:639-643]. Harter and Moore also had a study on asymptotic variances and covariances of maximum likelihood estimators, from censored samples, of the parameters of Weibull and Gamma distributions [22:557-570]. Woodruff and Moore used maximum likelihood estimation for the location and scale parameter estimates of the Weibull and they developed a modified goodness-of-fit test for the Weibull distribution [55:113-120]. Viviano [50] presented a thesis on goodness-of-fit for Gamma distribution by using the MLE's of the location and scale parameters. His study was the reference used to make the comparisons during this thesis effort. He also stated the shape parameter in the hypothesized null distribution but didn't use the M-D estimate of location. After the results of this study comparison tables and graphs were prepared to identify the effectiveness of each method.

Kappenman used a new procedure, in his study of "On a Method for Selecting a Distributional Model", that consists of computing the logarithm of the ratio of the maximized Gamma likelihood function to the maximized Weibull likelihood function and selecting the Gamma model if, and only if, this logarithm is positive. In this study the maximized Gamma likelihood function is obtained by replacing the parameters in the Gamma likelihood function with their maximum likelihood estimates [31:663-672]. Bowman and Shenton revisited the maximum likelihood estimators for the Gamma distribution and stated a new algorithm for the evaluation of the maximum likelihood estimators of the two-parameter Gamma density [8:697-710]. Recently, Rosaiah, Kantam and Narasimham studied on optimum class limits for ML estimation in two-parameter Gamma distribution from grouped data [41:1147-1179].

For the three parameter case, Cohen and Whitten presented an article which is concerned with modifications of both maximum likelihood and moment estimators for parameters of the three-parameter Gamma distribution. And for certain combinations of parameter values they got better estimators [13:197-216]. Bowman Shenton and Lam introduced a new algorithm for the solution of the three parameter model. They basically dealt with the simulation and estimation problems associated with the three-parameter Gamma density, and their study referred to samples of 50 or more [9:1147-1188].

The theory of maximum likelihood estimators provides no answers to the question concerning the properties of estimators for samples of finite size. The theory only guarantees that the maximum likelihood estimators approach the efficient ones as the sample size increases. "The reason that maximum likelihood estimates lack reasonable small-sample-size properties and do have reasonable large-sample-size properties is that the estimator is based upon trying to find the mode of a distribution by attempting to select the true value of a parameter. Estimators, in general, are designed to approach the true value rather than hitting it exactly right. Thus, one might expect

that other statistics such as the mean or the median of the a posteriori distribution should be better choices. The fact that the mode, mean and median values can vary significantly for small sample sizes leads to the undesirable characteristics of MLE for these cases. On the other hand, for large samples, the a posteriori distribution tends to become more or less independent of the a priori distribution, and the mode, mean and median statistics approach each other. Thus, it is not surprising that MLE has satisfactory asymptotic characteristics for large samples [15:135-136]."

Law and Kelton [32:370] listed that MLEs have several desirable statistical properties as follows:

1. For most of the common distributions, the MLE is unique; that is, $L(\hat{\theta})$ is strictly greater than $L(\theta)$ for any other value of $L(\theta)$.
2. Although MLEs need not be unbiased, in general, the asymptotic distribution (as $n \rightarrow \infty$) of $\hat{\theta}$ has mean equal to θ (see property 4 below).
3. MLEs are invariant; that is, if $\phi = h(\theta)$ for some function h , then the MLE of ϕ is $h(\hat{\theta})$. (Unbiasedness is not invariant.) For example, the variance of an $\text{expo}(\beta)$ random variable is β^2 , so the MLE of this variance is $[\bar{X}(n)]^2$.
4. MLEs are asymptotically normally distributed; that is, $\sqrt{n}(\hat{\theta} - \theta) \xrightarrow{D} N(0, \delta(\theta))$, where $\delta(\theta) = -n/E(d^2l/d\theta^2)$ (the expectation is with respect to X_i , assuming that X_i has the hypothesized distribution) and \xrightarrow{D} denotes convergence in distribution. Furthermore, if $\tilde{\theta}$ is any other estimator such that $\sqrt{n}(\tilde{\theta} - \theta) \xrightarrow{D} N(0, \sigma^2)$, then $\delta(\theta) \leq \sigma^2$. (Thus, MLEs are called best asymptotically normal.)
5. MLEs are strongly consistent; that is, $\lim_{n \rightarrow \infty} \hat{\theta} = \theta(w.p.1)$.

2.2.2 Minimum Distance Estimation. The minimum distance estimation method minimizes the *distance* between the cumulative distribution function (CDF)

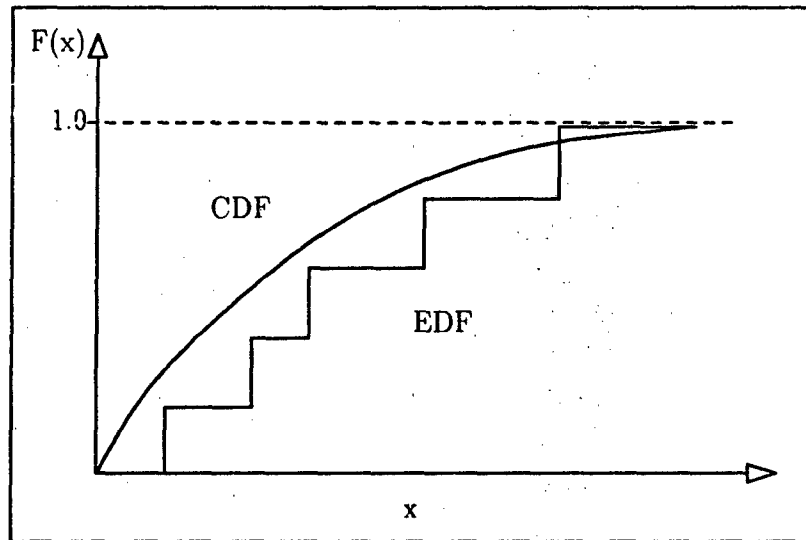


Figure 2.1. Cumulative and Empirical Distribution Functions. Figure also shows that parameters have a direct effect on the *distance* to be minimized.

values of the sample data and the empirical distribution function (EDF). Where "The EDF is step function, calculated from the sample, which estimates the population distribution function [46:97]." The CDF values come from a parameterized family of theoretical distribution functions. The EDF for n ordered data points $x_{(1)}, x_{(2)}, \dots, x_{(n)}$ is a step function given by

$$EDF(x) = \begin{cases} 0, & x < x_{(1)} \\ \frac{i}{n}, & x_{(i)} \leq x < x_{(i+1)}, \quad i = 1, \dots, (n-1) \\ 1, & x \geq x_{(n)} \end{cases} \quad (2.6)$$

The *distance* between the CDF and EDF is measured by a goodness-of-fit statistic. Method adjusts the parameters until the distance is minimized. The final result is the estimate of the parameters. The proposed new goodness-of-fit test will use a minimum distance estimator for the location parameter and MLE for the scale parameter.

Wolfowitz introduced the minimum distance (MD) method in the 1950's and showed that the minimum distance estimators are consistent. "In many stochastic structures where the distribution function (d.f.) depends continuously upon the parameters and d.f.'s of the chance variables in the structure, those parameters and d.f.'s which are identified (uniquely determined by the d.f. of the structure) can be strongly consistently estimated by the minimum distance method [46:75]." Wolfowitz pointed out the importance of his work with minimum distance estimators;

"A great utility of the minimum distance method is that, in a wide variety of problems, it will furnish super-consistent estimators even when classical methods, like the maximum likelihood method, fail to give consistent estimators [52]."

Wolfowitz's initial minimum distance estimator measured the distance between two cumulative distribution functions, $F_1(x)$ and $F_2(x)$, and the distance, $\delta(F_1, F_2)$ is defined by

$$\delta(F_1, F_2) = \sup_x |F_1(x) - F_2(x)| \quad (2.7)$$

Wolfowitz emphasized the applicability of this method for a broad range of distance techniques [52]Wol. Sahler studied conditions for the existence and consistency of MD estimates [42:13]. "Knusel examined the robustness of the MD method in 1969 and showed that MD estimators have similar robust properties as the maximum likelihood estimators [14:2-3]. Robustness refers to an estimation procedure that is good for a broad class of underlying models, but which is not necessarily the best estimating procedure for any one model [23:237]." Brean [5] has considered some D-estimators which require numerical integration and density estimation for their evaluation, but possess excellent asymptotic efficiency properties. Hobbs, Moore and James studied on the minimum distance estimation of the three parameters of the Gamma distribution. In their study, several new estimators were developed for the three-parameter Gamma distribution using minimum distance estimators for the location parameter with the remaining parameters determined by maximum likelihood estimators. They presented all relative s-efficiencies generated with sample size

of 8 and 20 and obtained the guaranteed minimum relative efficiency. Hobbs, Moore and Miller developed a 3-phase estimation technique and applied to the 3-parameter Weibull distribution. All of the estimators were better than the maximum likelihood estimators. And the technique using the Anderson-Darling statistic provided the best overall estimates of the parameters [24:495-496]. Charek, Moore and Coleman compared the minimum distance estimation with best linear unbiased estimation to determine which technique provides the most accurate estimates for location and scale parameters as applied to the 3-parameter pareto distribution. They found as a result that the best linear unbiased estimator provided more accurate estimates of location and scale than did the minimum distance estimators tested [10:1395-1407]. Gallagher and Moore proved that minimum distance estimation on the location parameter and maximum likelihood on the scale and shape parameters of the Weibull distribution is preferred over maximum likelihood estimates of all three parameters [18:575-580]. Recently Crown [14] studied the similar technique, which is used in this study, for the Weibull distribution. In most cases, minimizing the Anderson-Darling distance statistics to estimate the Weibull location parameter had more power than minimizing the Cramer-von Mises distance statistic. In his power study, the true null hypothesis achieved the expected level of significance.

Let $\Gamma = \{F_\theta(\cdot), \theta \in \Omega\}$ represent a parametric family of distributions, in this case the Gamma. $F_\theta(x)$ is an estimated distribution of the Gamma using the maximum likelihood and method of moments to determine the parameter estimates for the distribution function. Let $G_n(\cdot)$ denote the EDF based upon a random sample of size n , from the true distribution, $G(\cdot)$. $\delta(G_n, F_\theta)$ will denote the measure of discrepancy between the two distribution functions. (Gathered together from [42] [45] [39].)

2.2.2.1 Kolmogorov Distance.

$$D_\psi(G_n, F_\theta) = \sup_x |G_n(x) - F_\theta(x)| \psi[F_\theta(x)] \quad (2.8)$$

with interest given to $\psi(\cdot) \equiv 1$.

Stephens' computational formula;

$$\begin{aligned} D^+ &= \max [(i/n) - Z_i] \quad 1 \leq i \leq n \\ D^- &= \max [Z_i - (i-1)/n] \quad 1 \leq i \leq n \\ D &= \max [D^+, D^-] \end{aligned}$$

where $Z_i = F_\theta(x_i)$.

2.2.2.2 Cramer-von Mises Distance.

$$W_\psi^2(G_n, F_\theta) = \int_{-\infty}^{\infty} [G_n(x) - F_\theta(x)]^2 \psi[F_\theta(x)] dF_\theta(x) \quad (2.9)$$

with interest given to $\psi(\cdot) \equiv 1$.

Stephens' computational formula;

$$W^2 = \sum_{i=1}^n [Z_i - (2i-1)/2n]^2 + (1/12n) \quad (2.10)$$

where $Z_i = F_\theta(x_i)$.

2.2.2.3 Anderson-Darling Statistic. The Anderson-Darling statistic

is a spacial case of the Cramer-von Mises distance and it is one of the the more powerful empirical distribution function based tests of fit in a wide range of circumstances, where $\psi(u) = 1/[u(1-u)]$, $0 < u < 1$ and takes the form

$$A_n^2(G_n, F_\theta) = \int_{-\infty}^{\infty} [G_n(x) - F_\theta(x)]^2 [F_\theta(x)[1 - F_\theta(x)]]^{-1} dF_\theta(x) \quad (2.11)$$

Stephens' computational formula;

$$A^2 = - \left[\left[\sum_{i=1}^n (2i-1) [\ln Z_i + \ln(1 - Z_{n+1-i})] \right] / n \right] - n \quad (2.12)$$

where $Z_i = F_\theta(x_i)$.

Sinclair, Spurr and Ahmad worked on a modified A-D statistic that gives more weight to the larger and smaller data values [44:3677-3686]. Parr [39] indicated that four items were required to develop a minimum distance estimator: a set of data, a parametric model, a distance measure, and a minimization routine. In this thesis, data sets are generated by International Mathematical Statistics Library (IMSL) subroutine RNGAM. The parametric model is the three-parameter Gamma CDF. Anderson-Darling statistic is used as the distance measure. And subroutine MINDIS is used for the minimization routine.

2.2.3 Goodness-of-Fit Tests. A *goodness-of-fit test* is a statistical hypothesis test that is used to assess formally whether the observations X_1, X_2, \dots, X_n are an independent sample from a particular distribution with distribution function \hat{F} . That is, a goodness-of-fit test can be used to test the following null hypothesis:

H_0 : The X_i 's are i.i.d. random variables with distribution function \hat{F} .

We should remember that, failure to reject H_0 should not be interpreted as "accepting H_0 as being true." These test are often not very powerful for small to moderate sample size n ; that is, they are not very sensitive to subtle disagreements between the data and the fitted distribution. Instead, they should be regarded as a systematic approach for detecting fairly gross differences. On the other hand, if n is very large, then these test will almost always reject H_0 . Since H_0 is virtually never exactly true, even a minute departure from the hypothesized distribution will be detected for large n . This is an unfortunate property of these test, since it is usually sufficient to have a distribution that is nearly correct [32:380-382].

"Goodness-of-fit tests are rigorous way of checking to see whether the data we observed do, in fact, agree with the underlying probabilistic model we assumed for these data [43:348]." The oldest and probably the most commonly used goodness-

of-fit test is the chi-square test. In a simple case, it is a comparison of a histogram or line graph with the fitted density or mass function. If we divide the entire range of the fitted distribution into k intervals, and let N_j be the number of X_i 's in the j th interval and p_j be the expected proportion of the X_i 's that would fall in the j th interval; the test statistic is

$$\chi^2 = \sum_{j=1}^k \frac{(N_j - np_j)^2}{np_j} \quad (2.13)$$

We would expect χ^2 to be small if the fit is good. Because the chi-square test is easy to apply and can be used for both discrete and continuous cases, it has a wide use in data analysis. But in comparison with the others, it has lower power, especially when the sample size is small. The Kolmogorov-Smirnov, Cramer-von Mises and Anderson-Darling tests are well-suited to testing goodness-of-fit for continuous data and small sample sizes. They are more robust and their use is encouraged [36]. The equations related to these tests are presented in the previous sections. In the literature there are a lot of studies for the use and modifications of these tests for different distributions (See bibliography). Stephens [45] proposed the forms of the statistics used for the actual computations. Habib [20], Verril [49], and Chen [11] presented their dissertations on the goodness-of-fit tests for the censored data.

III. METHODOLOGY

3.1 Introduction

The Gamma distribution is the basis for this thesis research effort. So, this chapter will begin with the description of the Gamma distribution by way of its functions and properties. Next sections will cover the methodology steps, random deviate generation, Monte Carlo simulation, calculation of the critical values, the power study and verification and validation.

3.2 Discussion

3.2.1 Three Parameter Gamma Distribution. The use of Gamma distribution in different areas has been explained in the first chapter. In this section, density, cumulative and some other functions, some properties and relations with the other distributions are presented. Gamma density functions, for integer and non-integer shape values, are drawn in the following pages ($\beta = 1$ and $\delta = 10$).

3.2.1.1 Gamma Probability Density Function.

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.1)$$

where $\Gamma(\alpha)$ is the gamma function, defined by

$$\Gamma(z) = \int_0^x t^{z-1} e^{-t} dt \quad (3.2)$$

for any real number $z > 0$. Some properties of the gamma function: $\Gamma(z+1) = z\Gamma(z)$ for any $z > 0$, $\Gamma(k+1) = k!$ for any nonnegative integer k , $\Gamma(k + \frac{1}{2}) = \sqrt{\pi} \cdot 1 \cdot 3 \cdot 5 \cdots (2k-1)/2^k$ for any positive integer k , $\Gamma(1/2) = \sqrt{\pi}$.

3.2.1.2 Gamma Cumulative Distribution Function. If α is not integer, there is no closed form. If α is a positive integer, then

$$F(x) = \begin{cases} 1 - e^{-\frac{x}{\beta}} \sum_{j=0}^{\alpha-1} \frac{(\frac{x}{\beta})^j}{j!} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.3)$$

Shape parameter $\alpha > 0$, scale parameter $\beta > 0$.

3.2.1.3 MLE Estimators and Likelihood Function. The following equations must be satisfied

$$\ln \hat{\beta} + \Psi(\hat{\alpha}) = \frac{\sum_{i=1}^n \ln X_i}{n} \quad (3.4)$$

$$\hat{\alpha} \hat{\beta} = \bar{X}(n) \quad (3.5)$$

which could be solved numerically. [$\Psi(\hat{\alpha}) = \Gamma'(\hat{\alpha})/\Gamma(\hat{\alpha})$ and is called the digamma function.]

Likelihood function of the Gamma distribution with two parameters (α and β)

$$L(\alpha, \beta) = \frac{\beta^{-n\alpha} (\prod_{i=1}^n X_i)^{\alpha-1} \exp\left(-\frac{1}{\beta} \sum_{i=1}^n X_i\right)}{[\Gamma(\alpha)]^n} \quad (3.6)$$

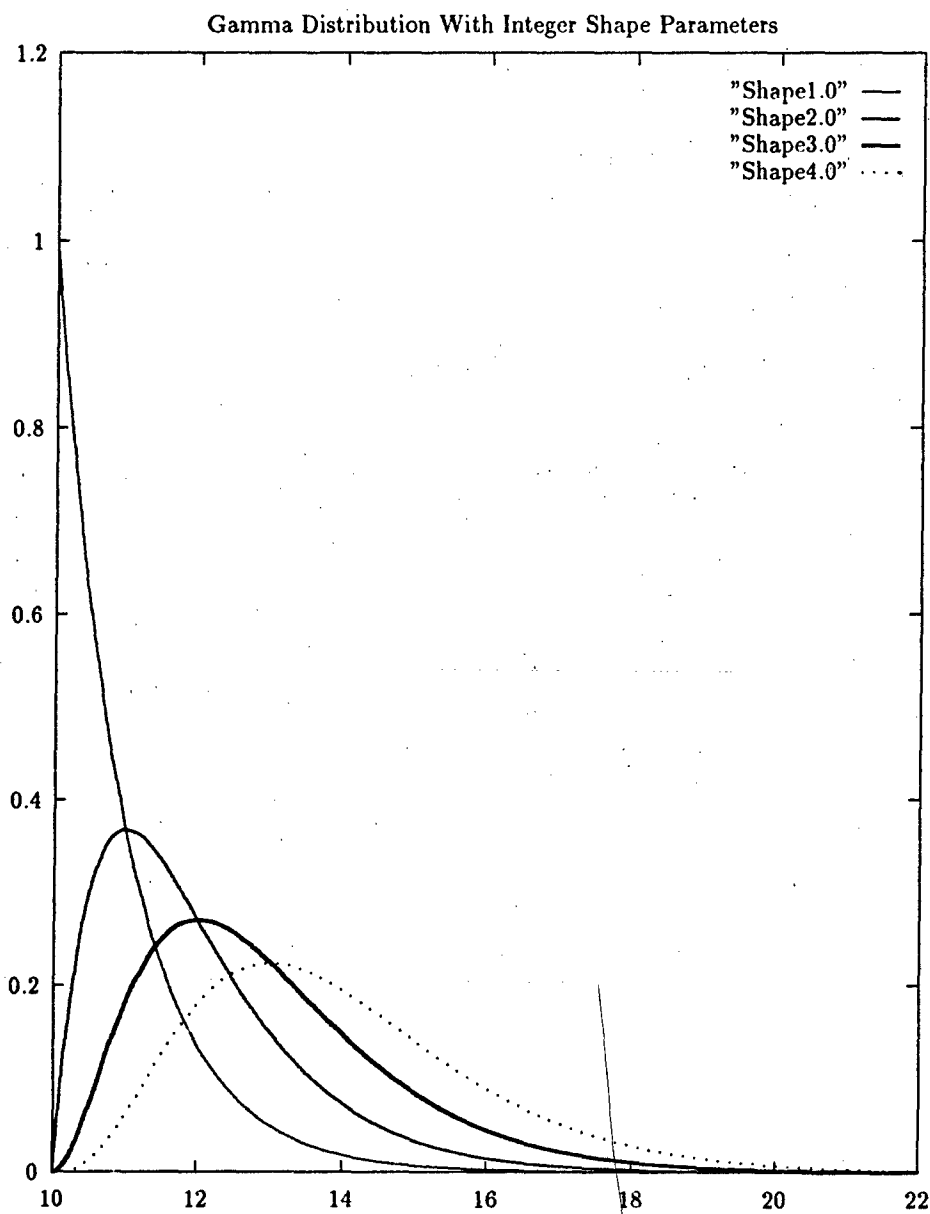


Figure 3.1. Gamma Distribution Density Functions, When α integer

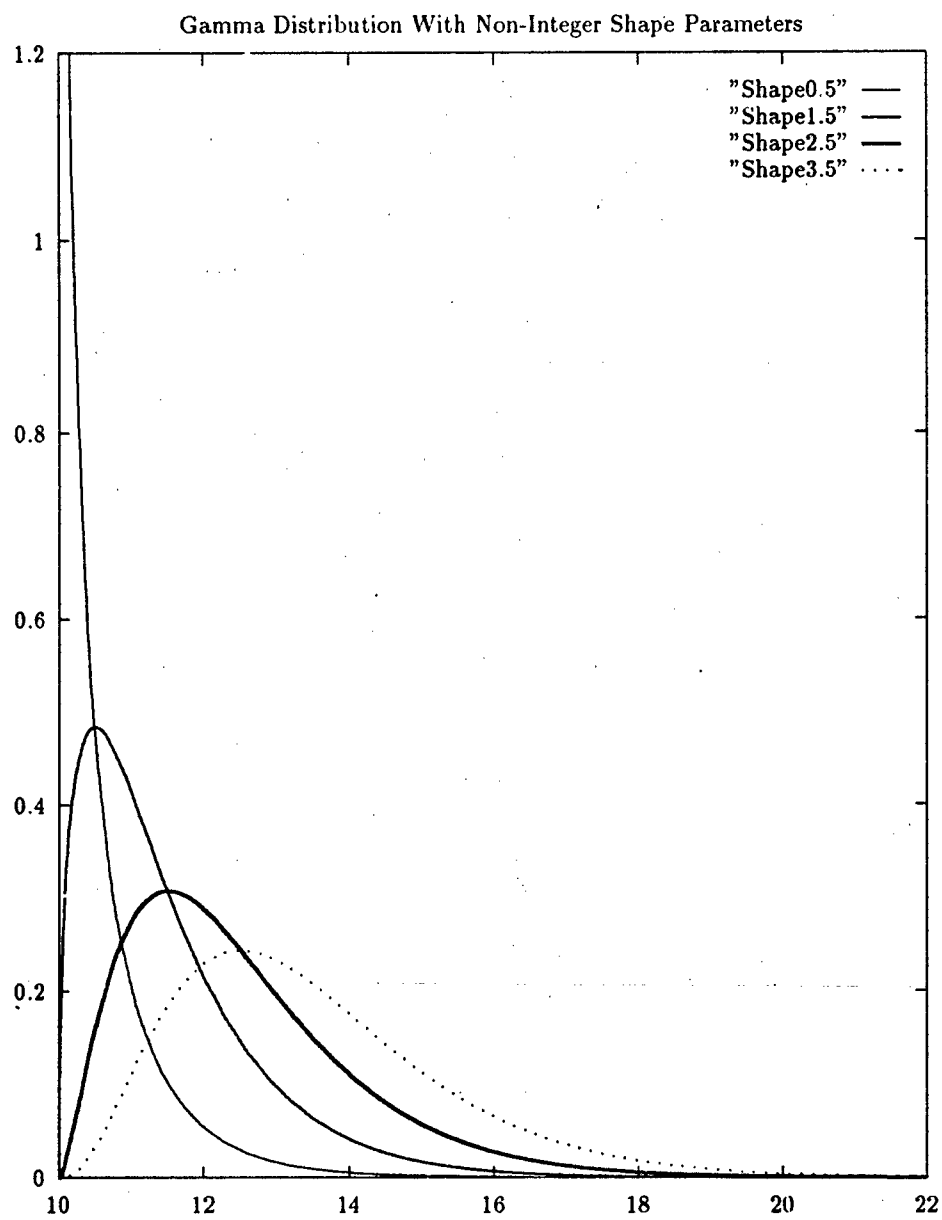


Figure 3.2. Gamma Distribution Density Functions, When α non-integer

3.2.1.4 Some Other Properties of Gamma Distribution.

1. The $\text{expo}(\beta)$ and $\text{gamma}(1, \beta)$ distributions are the same,
2. For a positive integer m , the $\text{gamma}(m, \beta)$ distribution is called the m -Erlang(β) distribution,
3. The chi-square distribution with k df is the same as the $\text{gamma}(k/2, 2)$ distribution,
4. If X_1, X_2, \dots, X_m are independent random variables with $X_i \sim \text{gamma}(\alpha_i, \beta)$, then $X_1 + X_2 + \dots + X_m \sim \text{gamma}(\alpha_1 + \alpha_2 + \dots + \alpha_m, \beta)$
5. If X_1 and X_2 are independent random variab. with $X_i \sim \text{gamma}(\alpha_i, \beta)$, then $X_1/(X_1 + X_2) \sim \text{beta}(\alpha_1, \alpha_2)$
6. $X \sim \text{gamma}(\alpha, \beta)$ if and only if $Y = 1/X$ has a Pearson type V distribution with shape and scale parameters α and $1/\beta$, denoted $\text{PT5}(\alpha, 1/\beta)$

7.

$$\lim_{x \rightarrow 0} f(x) = \begin{cases} \infty & \text{if } \alpha < 1 \\ \frac{1}{\beta} & \text{if } \alpha = 1 \\ 0 & \text{if } \alpha > 1 \end{cases}$$

(See [32:332]).

3.2.2 Random Deviate Generation. The first step will be to generate random samples 5,(5),40 from the Gamma distribution with scale, $\beta = 1$, location, $\delta = 10$ and shape, $\alpha = 0.5, (0.5), 4$. For the Gamma distribution function, there is no closed form for which we could obtain an inverse; however algorithms are available which can be used to generate random Gamma deviates. Stat \ Library IMSL Subroutine RNGAM has been used for the generation of the deviates. These standard deviates are converted to deviates having location $\delta = 10$ and scale $\beta = 1$.

This is done by using the transformation

$$z = \beta \cdot x + \delta \quad (3.7)$$

where x represents a standard random deviate generated by IMSL subroutine. This transformation is made to avoid a problem with the parameter estimating routine and to make better comparison with the study that is done recently.

Subroutine uses squared and halved normal deviates if $\alpha = 0.5$, exponential deviates if $\alpha = 1$, an acceptance-rejection method due to Ahrens if $\alpha < 1$ and a ten-region rejection procedure developed by Schmeiser and Lal if $\alpha > 1$ [26:1003].

3.2.3 Monte Carlo Simulation Method. One method for investigating the effects of nonlinearity or various other effects that are difficult to analyze otherwise is called the Monte Carlo method. This method involves the determination of the distributions of the various elements in a system, selection of the random sample of each element, and combining of these samples to obtain a measure of the system performance and reliability. The process of random selection and determination of the system effects are repeated a large number of times and, each repetition results in another different estimate of the system characteristic that is being measured. Beck and Arnold [4] say that, the Monte Carlo method can be used to investigate analytically the properties of a proposed estimation method. They give an algorithm to simulate a series of experiments on the computer. And they point out that the procedure is so flexible. The sample properties for any model, linear or nonlinear, and for any parameter values can be estimated. Also, you can estimate the effect of different probability distributions upon ordinary least squares estimation or other estimation methods.

"The usefulness of the Monte Carlo simulation method is based on the fact that the next best situation to having the probability distribution function of a certain

random quantity is to have a corresponding large population. The implementation of the method consists of numerically simulating a population corresponding to the random quantities in the physical problem, solving the deterministic problem associated with each member of that population, and obtaining a population corresponding to the random response quantities. This population can then be used to obtain statistics of the response variables. The method is a quite versatile mathematical tool capable of handling situations where all other methods fail to succeed [19:75]".

Its reliance on computers to simulate random processes is an important characteristic of the Monte Carlo method. The Monte Carlo approach is used in this thesis to generate the critical value tables for the Anderson-Darling goodness-of-fit technique. The approach is to observe random variates chosen so that they directly simulate the random process of the original problem. Basic steps of the simulation in this study:

1. Generate sample data from the selected underlying distribution.
2. Determine estimates of the parameters by using an estimation technique on each sample.
3. Compare the performance of the estimators.

Most of the studies until today show that 5000 repetition through the computations provide consistent results. So, this thesis uses 5000 repetitions to get consistent results and to save computer time.

3.2.4 Calculation of the Critical Values. The following procedure is used to generate the critical value tables for the modified goodness-of-fit test.

1. For a fixed sample size n and fixed shape parameter α , n standard random deviates are generated using computer subroutine. Standard deviates are converted

to the random deviates with location parameter $\delta = 10$ and scale parameter $\beta = 1$.

2. The n random deviates are ordered, $x_{(1)}, x_{(2)}, \dots, x_{(n)}$.
3. The ordered random deviates are used to estimate the maximum likelihood location and scale parameters.
4. The estimated scale and fixed shape parameter are used to estimate the minimum distance estimate of location, taking the previous step as an initial.
5. The MLE for scale is recalculated.
6. Estimated location and scale parameters and fixed shape parameter are used to determine the hypothesized distribution function.
7. The modified Anderson-Darling test statistic is calculated.
8. Steps 1 through 7 are repeated 5000 times.
9. The values of the statistic are ordered in ascending order and 80th, 85th, 90th, 95th and 99th percentiles are used as the critical values of the test.

3.2.5 Power Study. After the critical values have been calculated, a power study can be completed by using the different samples generated from different distributions. During this study IMSL random number generation subroutines are used for sample sizes 5, (5), 30. Once the samples have been generated, a random sample can be tested with the null hypothesis that the data is from a Gamma distribution with estimated parameters, versus the alternative hypothesis that the data is in fact from the distribution used in generating the samples. The alternate distributions used in the power study are:

1. Gamma distribution that is used in the null hypothesis is taken as the alternate distribution for the validation of the study

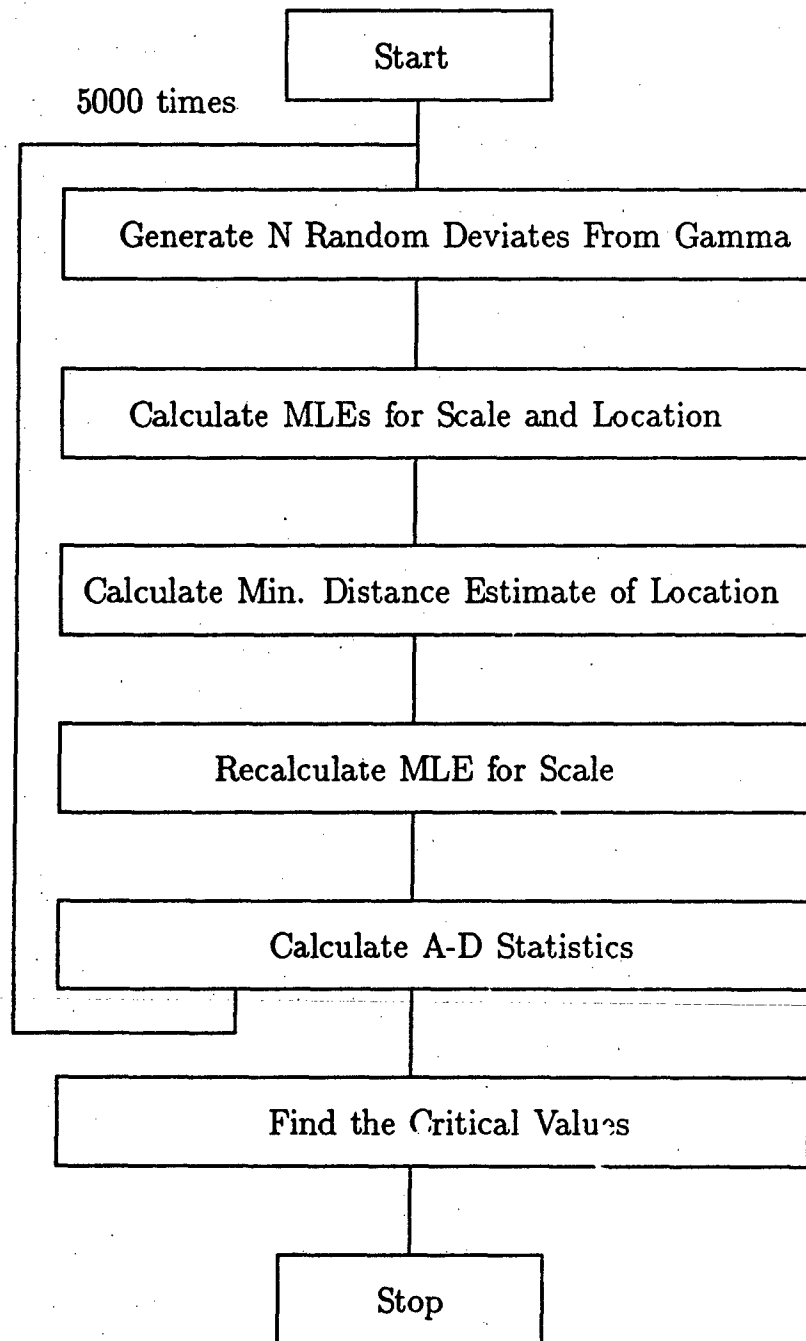


Figure 3.3. Flow Chart of Finding the Critical Values

2. Gamma, shape $\alpha = 2.5$
3. Gamma, shape $\alpha = 4.0$
4. Weibull, shape $\alpha = 2.0$
5. Weibull, shape $\alpha = 3.0$
6. Uniform, $U(10,15)$
7. Lognormal, $LNL(\omega = 0, \rho = 1)$
8. Lognormal, $LNL(\omega = 0, \rho = 2)$
9. Beta, $B(p = 1, q = 1)$
10. Beta, $B(p = 2, q = 2)$

Significance levels 0.01, 0.05, 0.10, 0.15, 0.20 are used for every distribution, during the study. The power investigation was conducted under the five null hypothesis; for the shape parameters of Gamma 0.5, 1.0, 1.5, 2.0, 3.0. For each of the sample sizes mentioned above, 5000 sample sets were generated for the alternative distributions. The modified A-D test statistic could be obtained using the minimum distance of location and maximum likelihood of scale parameter. The value of this statistic is compared to the critical values derived in this thesis. If the value of the test statistic is greater than the critical value, the null hypothesis is rejected. The total number of rejections are counted. The power of the test for a distribution can then be computed by dividing the number of rejections by the total number of trials, 5000. This power can be compared to the power of other known goodness-of-fit tests. The following chapters include a comparison with a study that use MLEs for both location and scale parameters.

3.2.6 Verification and Validation. Verification of the computer codes was accomplished by an extensive line by line check. And later on, during the study, results

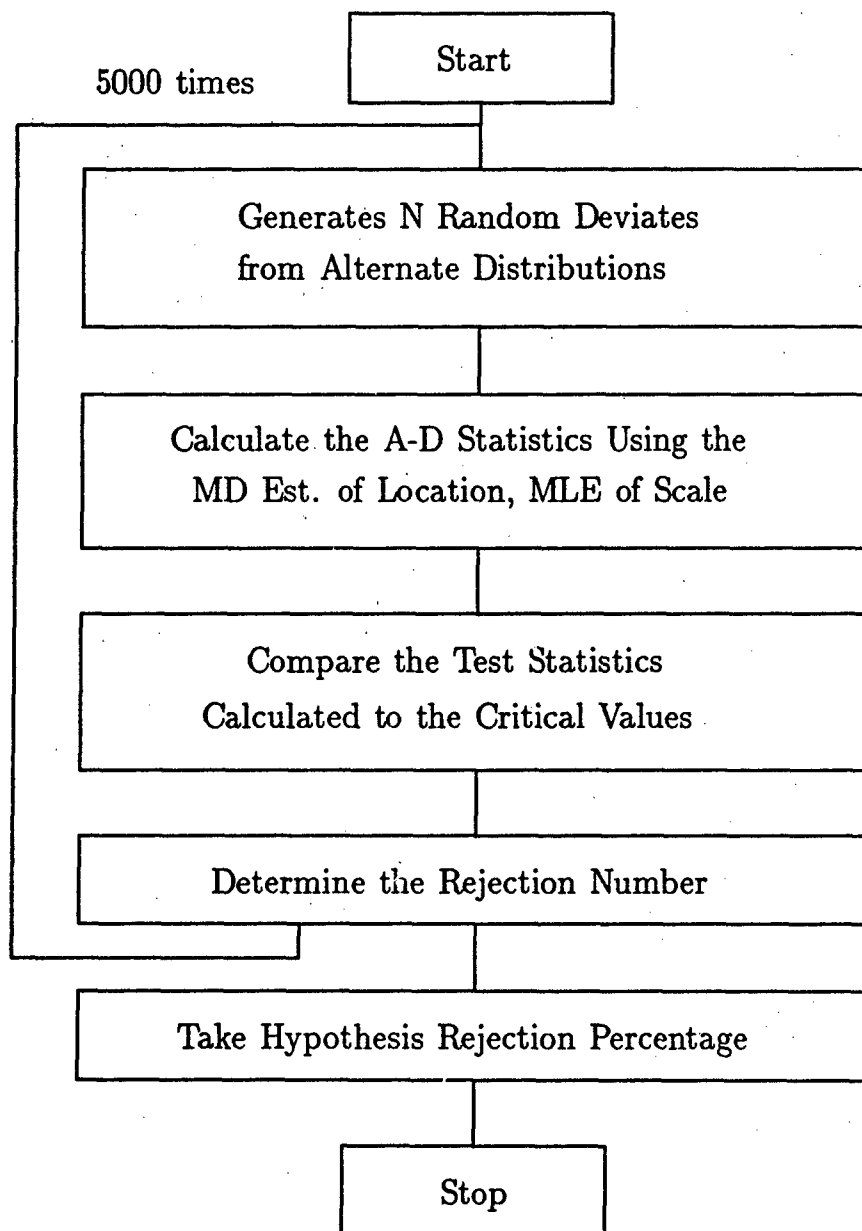


Figure 3.4. Flow Chart of Power Study

checked step by step with Mathcad. The same numbers observed. Also, the first column of the power study tables (Chp.4) are presented as a check point. Because in these columns, the null and alternate hypothesis are taken the same. The powers obtained the same as the significance levels; and this shows that our computer codes are valid.

IV. RESULTS

4.1 Anderson Darling Critical Values

The modified Anderson-Darling statistic is used to get the critical values for Gamma shape parameters 0.5, (0.5), 4.0, sample sizes 5, (5), 40 and significance levels 0.01, 0.05, 0.10, 0.15, 0.20. All the tables are given in the following pages. The critical values are increasing for every level of significance as the sample size increases. When the shape parameter is equal to two, the increase is relatively small. The use of the tables to perform a goodness-of-fit test follows these steps:

1. Determine the shape parameter and desired significance level.
2. From the data to be tested, calculate the minimum distance of location and maximum likelihood of scale.
3. From the appropriate table read the critical value.
4. Using the estimators you got at step 2, determine the estimated hypothesised distribution and calculate the A-D test statistic.
5. If the value obtained in step 4 is greater than the critical value found in step 3, then reject the hypothesized distribution; if not the hypothesized distribution can not be rejected.

4.1.1 Polynomial Functions for Critical Values. An investigation of the functional relationships between the critical values, sample size (n) and significance level (α) are examined when the n and α are variables. In this study, we had an

assumption that the shape parameter is known. Because of that I didn't include the shape parameter as a variable in the polynomials. But doing this increased the accuracy of the functions.

Stepwise regression method has been applied for every shape parameter and a different function is found for each shape. These functions are presented under the critical value table of each shape parameter. The use of the functions is very easy. For example, when shape is 0.5, sample size is $n = 20$ and significance level is $\alpha = 0.15$; if we put the values into the function the value we get is 2.89855 and reading the table gives 2.90586.

Anderson Darling Critical Values, Shape = 0.5

| 1-x | n=5 | n=10 | n=15 | n=20 | n=25 | n=30 | n=35 | n=40 |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|
| .20 | 1.00902 | 1.47727 | 2.08925 | 2.71001 | 3.37561 | 4.08965 | 4.71115 | 5.42480 |
| .15 | 1.07643 | 1.60555 | 2.24255 | 2.90586 | 3.58008 | 4.29668 | 4.95995 | 5.69822 |
| .10 | 1.16532 | 1.75183 | 2.45144 | 3.13793 | 3.84529 | 4.57785 | 5.24889 | 5.97833 |
| .05 | 1.30011 | 1.96825 | 2.71356 | 3.47167 | 4.22629 | 4.97994 | 5.72735 | 6.45425 |
| .01 | 1.53440 | 2.33949 | 3.22342 | 4.12514 | 4.90850 | 5.79500 | 6.56209 | 7.46431 |

Table 4.1. AD Critical Values for Shape = 0.5

Function of the critical values for Gamma shape= 0.5 (Confidence 0.9970)

$$CV = 0.88971 + 0.16412n - 9.86652\alpha - 0.20142n\alpha + 36.0303\alpha^2 \quad (4.1)$$

n =sample size and α =significance level.

Anderson Darling Critical Values, Shape = 1.0

| 1-x | n=5 | n=10 | n=15 | n=20 | n=25 | n=30 | n=35 | n=40 |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|
| .20 | 0.67434 | 0.79979 | 1.06126 | 1.23485 | 1.34786 | 1.49442 | 1.61408 | 1.79139 |
| .15 | 0.72691 | 0.89477 | 1.17946 | 1.35969 | 1.48064 | 1.65071 | 1.79560 | 1.97671 |
| .10 | 0.80129 | 1.01662 | 1.35270 | 1.53478 | 1.66627 | 1.84893 | 2.00810 | 2.19041 |
| .05 | 0.90419 | 1.21104 | 1.61627 | 1.80224 | 1.97667 | 2.17056 | 2.36925 | 2.54939 |
| .01 | 1.15274 | 1.60361 | 2.07047 | 2.31085 | 2.56099 | 2.90480 | 2.94969 | 3.35005 |

Table 4.2. AD Critical Values for Shape = 1.0

Function of the critical values for Gamma shape= 1.0 (Confidence 0.9841)

$$CV = 1.01518 + 0.0758n - 9.1686\alpha - 0.13433n\alpha - 0.0004447n^2 + 32.0377\alpha^2 \quad (4.2)$$

n =sample size and α =significance level.

Anderson Darling Critical Values, Shape = 1.5

| 1-x | n=5 | n=10 | n=15 | n=20 | n=25 | n=30 | n=35 | n=40 |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|
| .20 | 0.50377 | 0.54686 | 0.58554 | 0.61592 | 0.63824 | 0.68183 | 0.73557 | 0.77692 |
| .15 | 0.54783 | 0.60789 | 0.65253 | 0.68897 | 0.71556 | 0.76695 | 0.83240 | 0.86808 |
| .10 | 0.60541 | 0.68561 | 0.73672 | 0.78797 | 0.82113 | 0.89467 | 0.97704 | 0.99969 |
| .05 | 0.70041 | 0.81832 | 0.91014 | 0.93540 | 1.02782 | 1.12175 | 1.18746 | 1.23086 |
| .01 | 0.92867 | 1.15620 | 1.29766 | 1.40306 | 1.58143 | 1.63513 | 1.71618 | 1.80514 |

Table 4.3. AD Critical Values for Shape = 1.5

Function of the critical values for Gamma shape= 1.5 (Confidence 0.9653)

$$CV = 1.00672 + 0.02154n - 7.93372\alpha - 0.08029n\alpha + 27.9847\alpha^2 \quad (4.3)$$

n =sample size and α =significance level.

Anderson Darling Critical Values, Shape = 2.0

| 1-x | n=5 | n=10 | n=15 | n=20 | n=25 | n=30 | n=35 | n=40 |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|
| .20 | 0.50929 | 0.54297 | 0.55863 | 0.56356 | 0.57450 | 0.57793 | 0.57660 | 0.57901 |
| .15 | 0.55152 | 0.59577 | 0.60966 | 0.61738 | 0.62468 | 0.63553 | 0.63503 | 0.63493 |
| .10 | 0.60488 | 0.67107 | 0.68781 | 0.69833 | 0.70804 | 0.71413 | 0.72097 | 0.71351 |
| .05 | 0.70029 | 0.78159 | 0.81931 | 0.82063 | 0.85924 | 0.85021 | 0.85780 | 0.84227 |
| .01 | 0.90570 | 1.07655 | 1.10589 | 1.14766 | 1.19699 | 1.19670 | 1.15982 | 1.18563 |

Table 4.4. AD Critical Values for Shape = 2.0

Function of the critical values for Gamma shape= 2.0 (Confidence 0.9694)

$$CV = 0.98379 + 0.01225n - 6.27139\alpha - 0.01974n\alpha - 0.000156n^2 + 19.0327\alpha^2 \quad (4.4)$$

n =sample size and α =significance level.

Anderson Darling Critical Values, Shape = 2.5

| 1-x | n=5 | n=10 | n=15 | n=20 | n=25 | n=30 | n=35 | n=40 |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|
| .20 | 0.50629 | 0.53977 | 0.55370 | 0.56025 | 0.56529 | 0.57745 | 0.58444 | 0.58845 |
| .15 | 0.54916 | 0.59254 | 0.60418 | 0.62373 | 0.62350 | 0.63857 | 0.64605 | 0.64225 |
| .10 | 0.60302 | 0.66573 | 0.68381 | 0.69520 | 0.70133 | 0.72433 | 0.73063 | 0.72710 |
| .05 | 0.69025 | 0.79337 | 0.81971 | 0.82642 | 0.84876 | 0.87298 | 0.86622 | 0.86434 |
| .01 | 0.93684 | 1.06162 | 1.10351 | 1.13207 | 1.17845 | 1.26912 | 1.19942 | 1.15102 |

Table 4.5. AD Critical Values for Shape = 2.5

Function of the critical values for Gamma shape= 2.5 (Confidence 0.9693)

$$CV = 0.96959 + 0.01292n - 6.14333\alpha - 0.01919n\alpha - 0.000165n^2 + 18.4079\alpha^2 \quad (4.5)$$

n =sample size and α =significance level.

Anderson Darling Critical Values, Shape = 3.0

| 1-x | n=5 | n=10 | n=15 | n=20 | n=25 | n=30 | n=35 | n=40 |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|
| .20 | 0.50258 | 0.53550 | 0.55488 | 0.55806 | 0.57679 | 0.59287 | 0.60309 | 0.60372 |
| .15 | 0.54604 | 0.58881 | 0.60194 | 0.61939 | 0.63978 | 0.65661 | 0.65816 | 0.67220 |
| .10 | 0.60850 | 0.65970 | 0.67936 | 0.70162 | 0.71586 | 0.74064 | 0.74079 | 0.77029 |
| .05 | 0.70407 | 0.77737 | 0.81819 | 0.83309 | 0.85418 | 0.87436 | 0.90095 | 0.92948 |
| .01 | 0.93248 | 1.03612 | 1.12288 | 1.15538 | 1.22367 | 1.24678 | 1.19524 | 1.24453 |

Table 4.6. AD Critical Values for Shape = 3.0

Function of the critical values for Gamma shape= 3.0 (Confidence 0.9704)

$$CV = 0.96296 + 0.01366n - 6.28657\alpha - 0.03032n\alpha - 0.0001237n^2 + 19.6629\alpha^2 \quad (4.6)$$

n =sample size and α =significance level.

Anderson Darling Critical Values, Shape = 3.5

| 1-x | n=5 | n=10 | n=15 | n=20 | n=25 | n=30 | n=35 | n=40 |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|
| .20 | 0.51217 | 0.54502 | 0.56003 | 0.57641 | 0.58769 | 0.60593 | 0.61861 | 0.64073 |
| .15 | 0.55573 | 0.59714 | 0.61407 | 0.63206 | 0.65239 | 0.67972 | 0.69215 | 0.71433 |
| .10 | 0.61150 | 0.67150 | 0.68951 | 0.71965 | 0.73625 | 0.77042 | 0.78206 | 0.81352 |
| .05 | 0.71434 | 0.79616 | 0.82012 | 0.84707 | 0.87854 | 0.91123 | 0.95395 | 0.97282 |
| .01 | 0.97850 | 1.07822 | 1.17070 | 1.19057 | 1.28280 | 1.31273 | 1.29887 | 1.34198 |

Table 4.7. AD Critical Values for Shape = 3.5

Function of the critical values for Gamma shape= 3.5 (Confidence 0.9120)

$$CV = 0.0849n - 0.11941n\alpha - 0.00134n^2 \quad (4.7)$$

n =sample size and α =significance level.

Anderson Darling Critical Values, Shape = 4.0

| 1-x | n=5 | n=10 | n=15 | n=20 | n=25 | n=30 | n=35 | n=40 |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|
| .20 | 0.51501 | 0.54896 | 0.56905 | 0.59174 | 0.60366 | 0.61896 | 0.64603 | 0.66007 |
| .15 | 0.55791 | 0.60454 | 0.62058 | 0.65473 | 0.66588 | 0.68954 | 0.71720 | 0.73033 |
| .10 | 0.62170 | 0.68687 | 0.70921 | 0.74029 | 0.76065 | 0.79391 | 0.81386 | 0.84153 |
| .05 | 0.73945 | 0.80865 | 0.82834 | 0.88723 | 0.90698 | 0.94380 | 0.96680 | 1.01297 |
| .01 | 1.03442 | 1.13090 | 1.15191 | 1.22604 | 1.26343 | 1.31947 | 1.39282 | 1.42848 |

Table 4.8. AD Critical Values for Shape = 4.0

Function of the critical values for Gamma shape= 4.0 (Confidence 0.9714)

$$CV = 1.05828 + 0.01003n - 6.95325\alpha - 0.03406n\alpha + 21.8651\alpha^2 \quad (4.8)$$

n =sample size and α =significance level.

4.2 Power Study

In this section the results of the power study will be shown. The tables are presented in the following pages. In general, as expected, the powers are increasing when the sample size increase and an increase always occurred with the increase of the significance level. Certainly, there are some exceptions. For small shape parameters of Gamma, the power on lognormal is not high. But this is not something unexpected, if you look at their shapes on a graph, they look so similar to each others. Also, powers come down for the similar Gamma's, especially for greater values of shape (the distribution loses its skewness). So, if you take a 2.0 or greater shape valued Gamma as the null hypothesis and another Gamma with 2.0 or greater shape value, it is clear that the power will be small.

An interesting result came with the lognormal $LNL(0,1)$ when the null hypothesis is a Gamma with shape equals 0.5 or 1.0; the increase on sample size cause the power to go down. As mentioned before, the power is not good for this alternate distribution because of their similarity. Another observation for Gamma shape 0.5, the power goes down fast for the significance level is equal to 0.01. For Gamma shape 1.5 we got low power for $LNL(0,1)$ but excellent power for $LNL(0,2)$, even when the significance level is small. For the same null hypothesis, also we got pretty good power on the beta distribution. When Gamma shape is equal to 2.0 powers are good for all alternates, especially for lognormal and beta distributions. Taking the null hypothesis Gamma with shape 3.0, it results in good power for lognormal and beta again, especially for small sample sizes we get the highest powers in the study.

The graphs followed by the power tables are presented to see the differences clearly. Also, the last two tables and graphs are prepared for the verification of the effectiveness of the study. A previous study has been compared with the modified one for two shape parameters (1.5 and 4.0) over 104 cases. The previous study was presented by Viviano [50] who used MLEs for both location and scale parameters. The results of this thesis effort are compared with that one to see whether we can

get better results. 104 cases were compared and it is found that, in 56% of the cases the new test is powerful, in 21% of the cases powers are the same and in 23% of the cases the previous study gets slightly more power. Especially for small sample sizes of lognormal and all samples of beta distribution, the modified test gets extremely more powerful results than the previous one (i.e. 0.044 against 0.995). Also, for Gamma shape=1.5 against Weibull shape= 2.0 and 3.0, the modified study appeared with greater difference over the previous study.

Two graphs are presented with the average powers for each method. To do that, the powers are ordered separately and plotted. It is clearly seen on the graphs that modified A-D test statistic provides better results.

H_0 : Gamma with Shape = 0.5; H_a : Another Distribution

| Sample Size | 1-x | Gamma Shape=0.5 | Gamma Shape=2.5 | Gamma Shape=4.0 | Weibull Shape=2.0 | Weibull Shape=3.0 |
|-------------|-----|-----------------|-----------------|-----------------|-------------------|-------------------|
| 5 | .20 | 0.19992 | 0.18673 | 0.15974 | 0.32967 | 0.39804 |
| | .15 | 0.14994 | 0.13994 | 0.11375 | 0.25710 | 0.32527 |
| | .10 | 0.09996 | 0.08896 | 0.06917 | 0.17673 | 0.23571 |
| | .05 | 0.04998 | 0.04138 | 0.03019 | 0.08976 | 0.13315 |
| | .01 | 0.01000 | 0.00460 | 0.00320 | 0.01599 | 0.02319 |
| 10 | .20 | 0.19432 | 0.29508 | 0.41503 | 0.44402 | 0.60716 |
| | .15 | 0.14494 | 0.22351 | 0.33727 | 0.35886 | 0.52899 |
| | .10 | 0.09516 | 0.16074 | 0.25410 | 0.26869 | 0.44442 |
| | .05 | 0.04858 | 0.08457 | 0.15294 | 0.17013 | 0.31407 |
| | .01 | 0.01220 | 0.02359 | 0.04858 | 0.05378 | 0.13015 |
| 15 | .20 | 0.19932 | 0.33207 | 0.48121 | 0.54778 | 0.76030 |
| | .15 | 0.14374 | 0.26190 | 0.41004 | 0.47301 | 0.70432 |
| | .10 | 0.08996 | 0.17993 | 0.31427 | 0.36246 | 0.61136 |
| | .05 | 0.05178 | 0.11176 | 0.21292 | 0.24210 | 0.48920 |
| | .01 | 0.01299 | 0.03319 | 0.07417 | 0.08856 | 0.26709 |
| 20 | .20 | 0.19712 | 0.37425 | 0.57577 | 0.63175 | 0.85406 |
| | .15 | 0.14854 | 0.29388 | 0.49200 | 0.55378 | 0.80848 |
| | .10 | 0.09896 | 0.21991 | 0.40364 | 0.45462 | 0.74050 |
| | .05 | 0.05118 | 0.12455 | 0.28129 | 0.31867 | 0.62235 |
| | .01 | 0.00920 | 0.03179 | 0.10936 | 0.11915 | 0.38645 |
| 25 | .20 | 0.20291 | 0.41044 | 0.63695 | 0.70792 | 0.90584 |
| | .15 | 0.15074 | 0.34106 | 0.56058 | 0.63135 | 0.87605 |
| | .10 | 0.09416 | 0.24790 | 0.46761 | 0.54018 | 0.82527 |
| | .05 | 0.04998 | 0.15574 | 0.33886 | 0.40704 | 0.73671 |
| | .01 | 0.01160 | 0.05018 | 0.15034 | 0.19412 | 0.53219 |
| 30 | .20 | 0.17753 | 0.41863 | 0.67833 | 0.74190 | 0.94082 |
| | .15 | 0.13855 | 0.35726 | 0.61735 | 0.68413 | 0.92223 |
| | .10 | 0.09076 | 0.27129 | 0.53219 | 0.60256 | 0.88685 |
| | .05 | 0.04638 | 0.17273 | 0.41264 | 0.47541 | 0.81847 |
| | .01 | 0.00760 | 0.04938 | 0.19572 | 0.22511 | 0.62235 |

Table 4.9. Power Test for Gamma, Shape = 0.5

H_0 : Gamma with Shape = 0.5; H_a : Another Distribution

| Sample Size | 1-x | Uniform (10,15) | Lognorm. $\omega = 0, \rho = 1$ | Lognorm. $\omega = 0, \rho = 2$ | Beta $p=1, q=1$ | Beta $p=2, q=2$ |
|-------------|-----|-----------------|---------------------------------|---------------------------------|-----------------|-----------------|
| 5 | .20 | 0.24350 | 0.12115 | 0.08157 | 0.37485 | 0.38305 |
| | .15 | 0.18613 | 0.08477 | 0.05918 | 0.30288 | 0.31048 |
| | .10 | 0.12215 | 0.05478 | 0.04258 | 0.22411 | 0.22491 |
| | .05 | 0.05118 | 0.02699 | 0.02619 | 0.12975 | 0.12455 |
| | .01 | 0.00480 | 0.00340 | 0.01379 | 0.02679 | 0.01859 |
| 10 | .20 | 0.53159 | 0.06357 | 0.13015 | 0.46901 | 0.57897 |
| | .15 | 0.44282 | 0.04078 | 0.10856 | 0.38805 | 0.48581 |
| | .10 | 0.34946 | 0.02539 | 0.08936 | 0.29728 | 0.39144 |
| | .05 | 0.23111 | 0.01100 | 0.06397 | 0.19812 | 0.26070 |
| | .01 | 0.08717 | 0.00120 | 0.04058 | 0.07617 | 0.0576 |
| 15 | .20 | 0.56377 | 0.02999 | 0.14214 | 0.54078 | 0.71491 |
| | .15 | 0.49140 | 0.01999 | 0.12395 | 0.46581 | 0.64494 |
| | .10 | 0.39044 | 0.01260 | 0.10156 | 0.36226 | 0.54218 |
| | .05 | 0.27009 | 0.00560 | 0.08077 | 0.24670 | 0.40104 |
| | .01 | 0.10976 | 0.00160 | 0.05578 | 0.09916 | 0.18693 |
| 20 | .20 | 0.61276 | 0.02099 | 0.16094 | 0.59576 | 0.81887 |
| | .15 | 0.52639 | 0.01160 | 0.13615 | 0.51459 | 0.76010 |
| | .10 | 0.43743 | 0.00480 | 0.11455 | 0.42083 | 0.67433 |
| | .05 | 0.31008 | 0.00200 | 0.08936 | 0.29628 | 0.53918 |
| | .01 | 0.12675 | 0.00040 | 0.05558 | 0.11515 | 0.27649 |
| 25 | .20 | 0.66114 | 0.01519 | 0.17053 | 0.63815 | 0.88245 |
| | .15 | 0.58377 | 0.00920 | 0.15034 | 0.56877 | 0.83567 |
| | .10 | 0.48021 | 0.00480 | 0.12835 | 0.46362 | 0.77249 |
| | .05 | 0.34706 | 0.00140 | 0.09936 | 0.32007 | 0.66194 |
| | .01 | 0.15854 | 0.00005 | 0.06697 | 0.14514 | 0.40584 |
| 30 | .20 | 0.66034 | 0.00720 | 0.16733 | 0.66334 | 0.91004 |
| | .15 | 0.58757 | 0.00400 | 0.14654 | 0.59436 | 0.88085 |
| | .10 | 0.49380 | 0.00200 | 0.12395 | 0.50200 | 0.82947 |
| | .05 | 0.36585 | 0.00120 | 0.10016 | 0.37085 | 0.72771 |
| | .01 | 0.16114 | 0.00020 | 0.06437 | 0.15594 | 0.48341 |

Table 4.10. Power Test for Gamma, Shape = 0.5 (Continue)

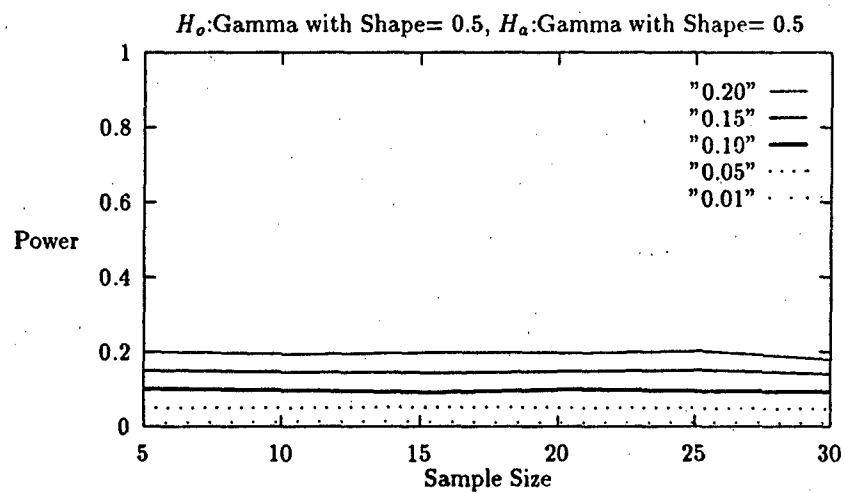


Figure 4.1. Power Study, Gamma Shape=0.5, Gamma Shape=0.5

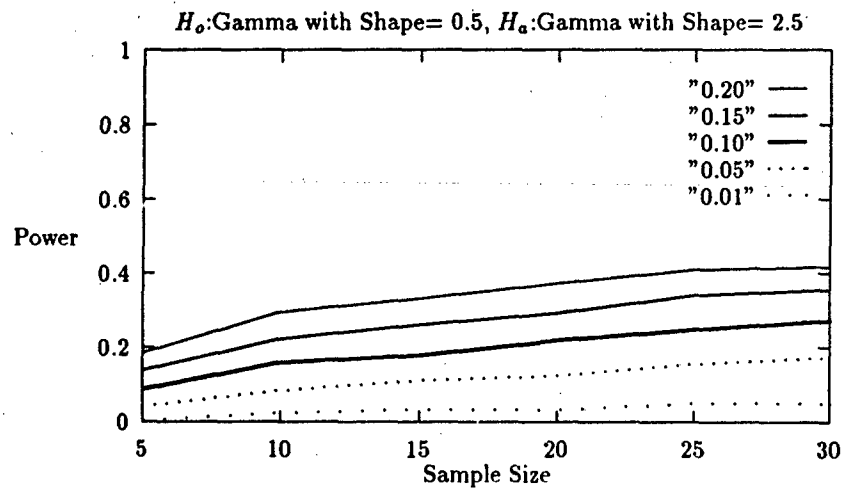


Figure 4.2. Power Study, Gamma Shape=0.5, Gamma Shape=2.5

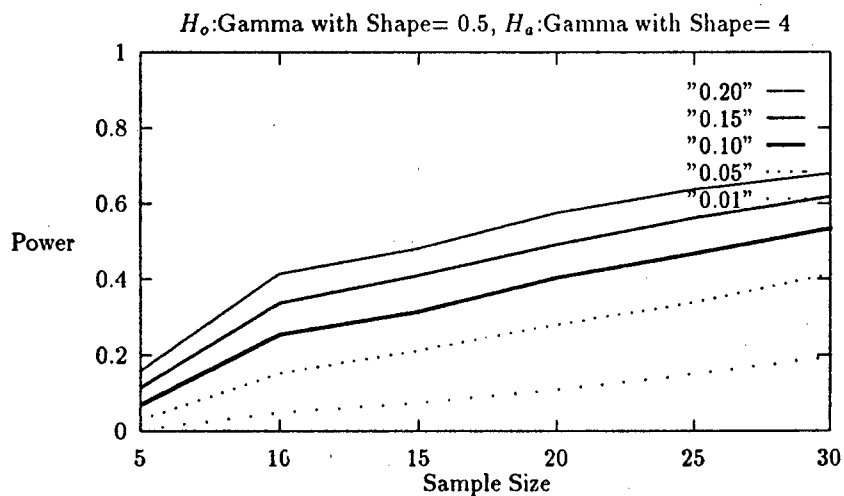


Figure 4.3. Power Study, Gamma Shape=0.5, Gamma Shape=4

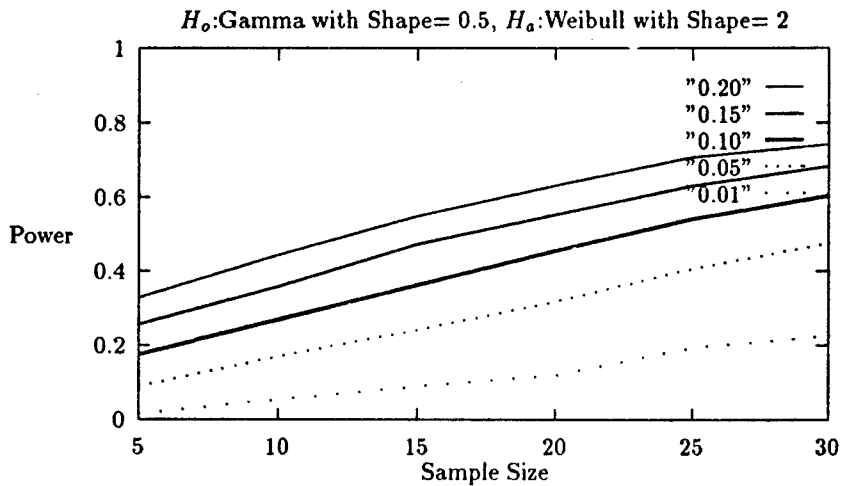


Figure 4.4. Power Study, Gamma Shape=0.5, Weibull Shape=2

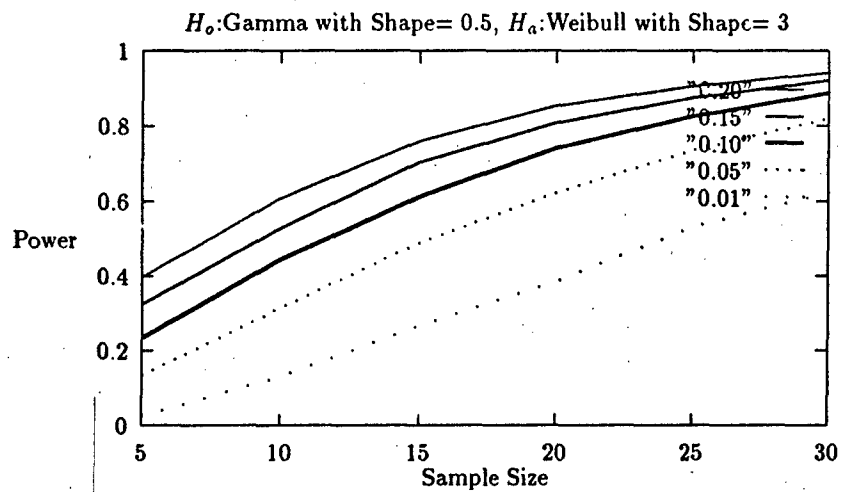


Figure 4.5. Power Study, Gamma Shape=0.5, Weibull Shape=3

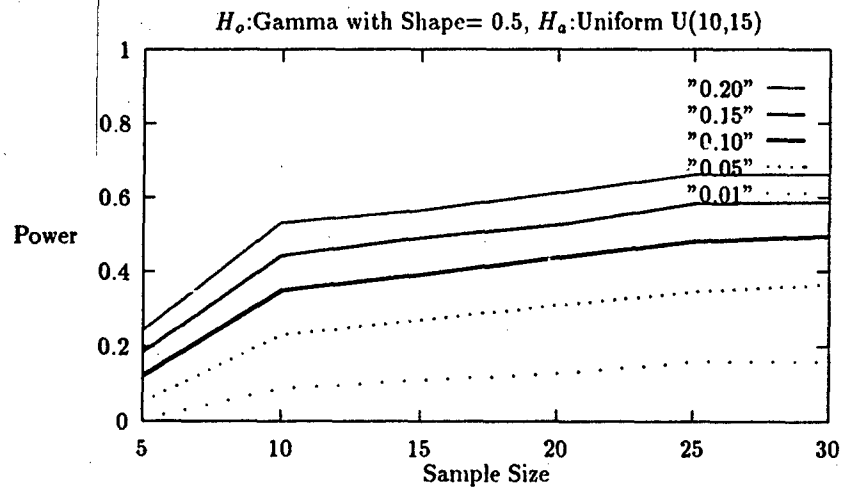


Figure 4.6. Power Study, Gamma Shape=0.5, Uniform U(10,15)

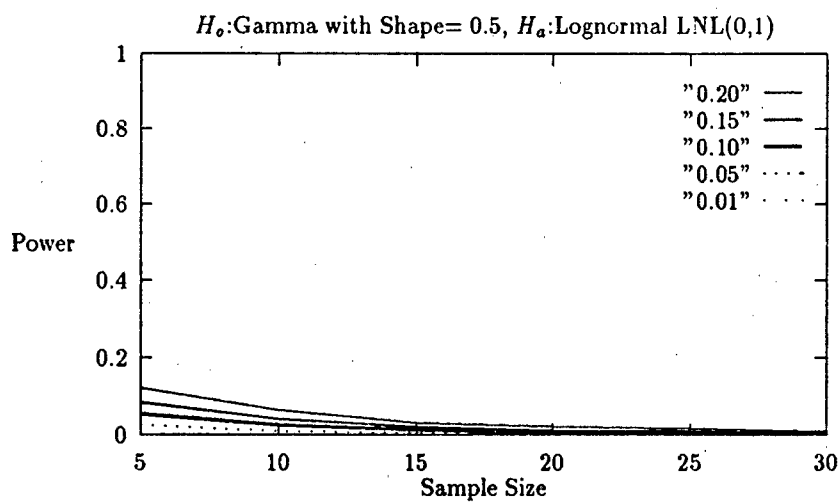


Figure 4.7. Power Study, Gamma Shape=0.5, Lognormal LNL(0,1)

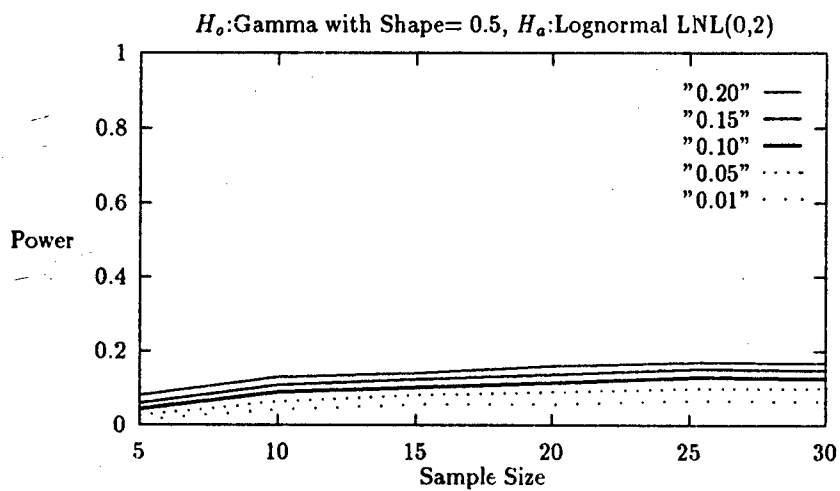


Figure 4.8. Power Study, Gamma Shape=0.5, Lognormal LNL(0,2)

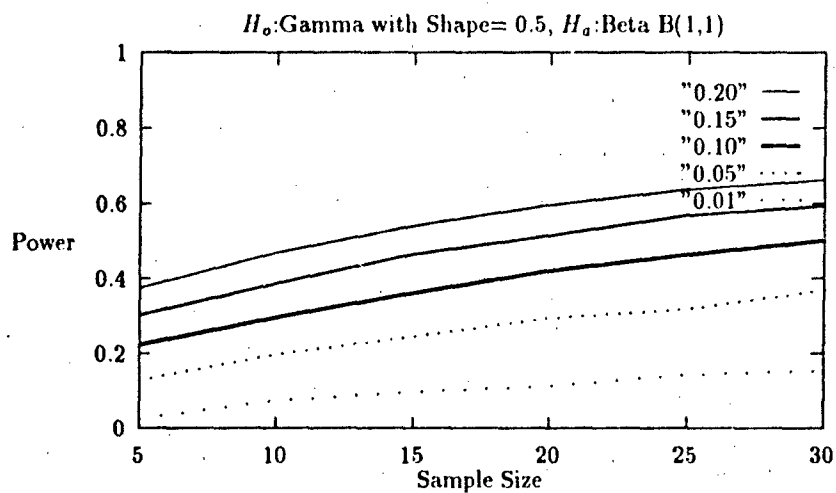


Figure 4.9. Power Study, Gamma Shape=0.5, Beta B(1,1)

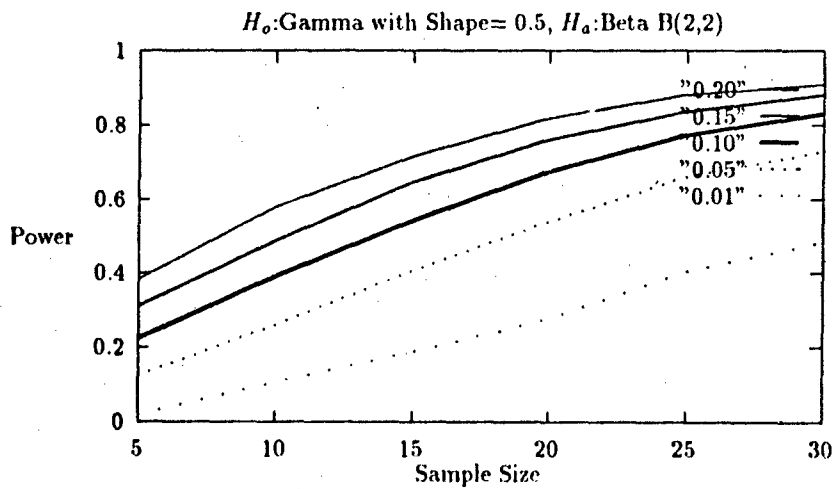


Figure 4.10. Power Study. Gamma Shape=0.5, Beta B(2,2)

H_0 : Gamma with Shape = 1.0; H_a : Another Distribution

| Sample Size | 1-x | Gamma Shape=1.0 | Gamma Shape=2.5 | Gamma Shape=4.0 | Weibull Shape=2.0 | Weibull Shape=3.0 |
|-------------|-----|-----------------|-----------------|-----------------|-------------------|-------------------|
| 5 | .20 | 0.19992 | 0.21272 | 0.23531 | 0.37965 | 0.55258 |
| | .15 | 0.14994 | 0.16194 | 0.18353 | 0.31967 | 0.48861 |
| | .10 | 0.09996 | 0.10216 | 0.12015 | 0.24650 | 0.41064 |
| | .05 | 0.04998 | 0.05238 | 0.06118 | 0.17013 | 0.32626 |
| | .01 | 0.01000 | 0.00860 | 0.01120 | 0.07897 | 0.19552 |
| 10 | .20 | 0.20692 | 0.26250 | 0.34106 | 0.68673 | 0.88085 |
| | .15 | 0.14894 | 0.19592 | 0.26310 | 0.60516 | 0.81248 |
| | .10 | 0.09576 | 0.13015 | 0.19452 | 0.50220 | 0.71691 |
| | .05 | 0.04838 | 0.06897 | 0.10956 | 0.33886 | 0.56078 |
| | .01 | 0.00960 | 0.01359 | 0.02679 | 0.12835 | 0.27229 |
| 15 | .20 | 0.18753 | 0.23471 | 0.35886 | 0.61415 | 0.81368 |
| | .15 | 0.14594 | 0.18713 | 0.28549 | 0.51519 | 0.74790 |
| | .10 | 0.09216 | 0.12075 | 0.20332 | 0.39204 | 0.64734 |
| | .05 | 0.04298 | 0.05958 | 0.10816 | 0.23771 | 0.49120 |
| | .01 | 0.01099 | 0.01359 | 0.03319 | 0.08117 | 0.25490 |
| 20 | .20 | 0.19792 | 0.31847 | 0.46961 | 0.60736 | 0.85366 |
| | .15 | 0.14154 | 0.25970 | 0.40244 | 0.53459 | 0.80868 |
| | .10 | 0.09376 | 0.18533 | 0.32087 | 0.43403 | 0.74050 |
| | .05 | 0.04658 | 0.10376 | 0.22771 | 0.29848 | 0.61395 |
| | .01 | 0.01219 | 0.02999 | 0.08856 | 0.11216 | 0.38725 |
| 25 | .20 | 0.20692 | 0.37045 | 0.58916 | 0.67893 | 0.90964 |
| | .15 | 0.15434 | 0.29968 | 0.51659 | 0.60836 | 0.87645 |
| | .10 | 0.10116 | 0.22291 | 0.42863 | 0.50880 | 0.82787 |
| | .05 | 0.05318 | 0.12895 | 0.29148 | 0.36825 | 0.72291 |
| | .01 | 0.01000 | 0.03359 | 0.12195 | 0.15694 | 0.50640 |
| 30 | .20 | 0.19492 | 0.37185 | 0.64454 | 0.73331 | 0.94862 |
| | .15 | 0.14414 | 0.31048 | 0.57297 | 0.66613 | 0.92423 |
| | .10 | 0.10156 | 0.22651 | 0.48940 | 0.57437 | 0.88345 |
| | .05 | 0.04738 | 0.13835 | 0.36106 | 0.43443 | 0.81128 |
| | .01 | 0.00740 | 0.03099 | 0.13954 | 0.17733 | 0.56757 |

Table 4.11. Power Test for Gamma, Shape = 1.0

H_0 : Gamma with Shape = 1.0; H_a : Another Distribution

| Sample Size | 1-x | Uniform (10,15) | Lognorm. $\omega = 0, \rho = 1$ | Lognorm. $\omega = 0, \rho = 2$ | Beta $p=1, q=1$ | Beta $p=2, q=2$ |
|-------------|-----|-----------------|---------------------------------|---------------------------------|-----------------|-----------------|
| 5 | .20 | 0.37265 | 0.19912 | 0.38984 | 0.58717 | 0.73271 |
| | .15 | 0.30368 | 0.15454 | 0.33647 | 0.52039 | 0.69112 |
| | .10 | 0.21791 | 0.10316 | 0.28049 | 0.43603 | 0.63155 |
| | .05 | 0.12095 | 0.06397 | 0.22151 | 0.34306 | 0.57257 |
| | .01 | 0.02399 | 0.01659 | 0.14214 | 0.20412 | 0.41663 |
| 10 | .20 | 0.54838 | 0.15654 | 0.57157 | 0.84266 | 0.89544 |
| | .15 | 0.45742 | 0.11236 | 0.51220 | 0.76449 | 0.82527 |
| | .10 | 0.35006 | 0.07537 | 0.43823 | 0.64914 | 0.71811 |
| | .05 | 0.21391 | 0.04138 | 0.35466 | 0.46861 | 0.52979 |
| | .01 | 0.06717 | 0.01040 | 0.24430 | 0.18932 | 0.22491 |
| 15 | .20 | 0.55758 | 0.10636 | 0.65854 | 0.71391 | 0.81767 |
| | .15 | 0.47021 | 0.07857 | 0.60156 | 0.61815 | 0.74570 |
| | .10 | 0.35626 | 0.05178 | 0.54078 | 0.47861 | 0.62555 |
| | .05 | 0.20912 | 0.02659 | 0.44602 | 0.30788 | 0.44742 |
| | .01 | 0.07577 | 0.00920 | 0.32727 | 0.11735 | 0.19492 |
| 20 | .20 | 0.65614 | 0.08097 | 0.75910 | 0.72051 | 0.84406 |
| | .15 | 0.58297 | 0.06178 | 0.71831 | 0.63815 | 0.79008 |
| | .10 | 0.48041 | 0.04518 | 0.66513 | 0.52899 | 0.69312 |
| | .05 | 0.34086 | 0.02719 | 0.58097 | 0.36346 | 0.54398 |
| | .01 | 0.14234 | 0.00980 | 0.44102 | 0.15474 | 0.29308 |
| 25 | .20 | 0.74630 | 0.08457 | 0.84006 | 0.75990 | 0.89444 |
| | .15 | 0.67393 | 0.06258 | 0.80808 | 0.68593 | 0.85326 |
| | .10 | 0.57297 | 0.04018 | 0.76649 | 0.57737 | 0.78329 |
| | .05 | 0.40484 | 0.02059 | 0.68753 | 0.41064 | 0.64674 |
| | .01 | 0.18473 | 0.00820 | 0.54078 | 0.17153 | 0.38205 |
| 30 | .20 | 0.77609 | 0.07557 | 0.88805 | 0.78009 | 0.93323 |
| | .15 | 0.69852 | 0.05738 | 0.85966 | 0.69152 | 0.90004 |
| | .10 | 0.59736 | 0.04098 | 0.82227 | 0.58437 | 0.84746 |
| | .05 | 0.44662 | 0.02479 | 0.75330 | 0.42803 | 0.74390 |
| | .01 | 0.17913 | 0.00800 | 0.60216 | 0.15854 | 0.43823 |

Table 4.12. Power Test for Gamma, Shape = 1.0 (Continue)

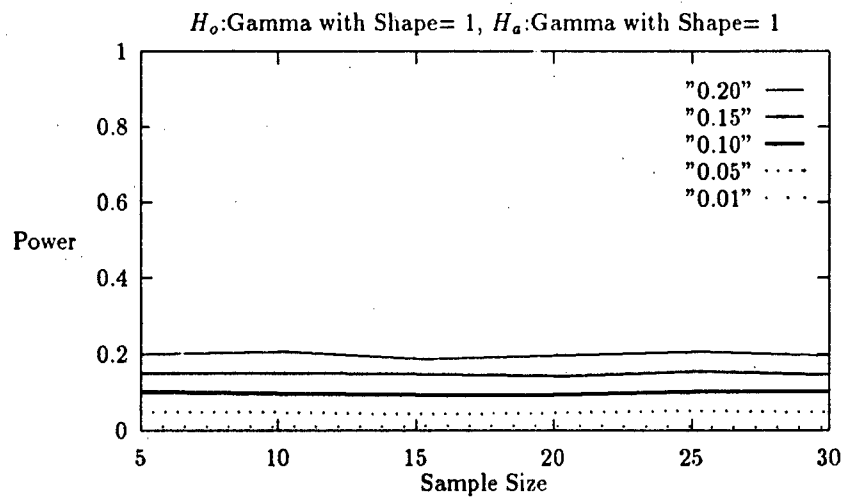


Figure 4.11. Power Study, Gamma Shape=1, Gamma Shape=1

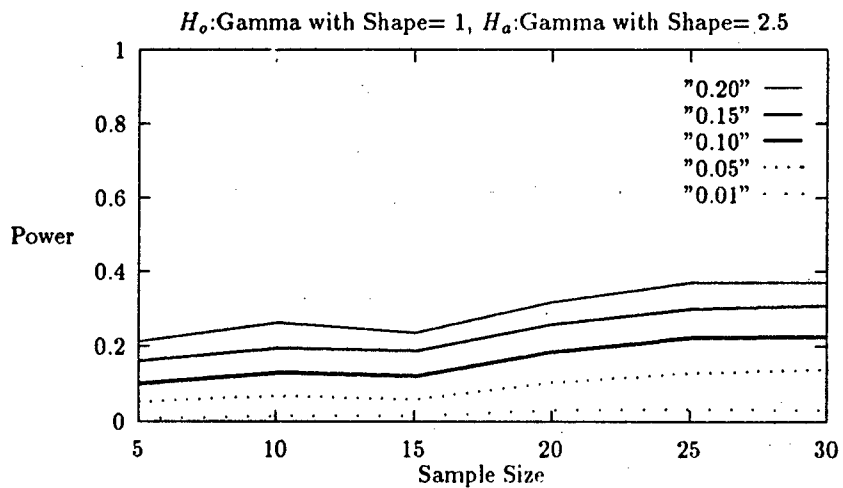


Figure 4.12. Power Study, Gamma Shape=1, Gamma Shape=2.5

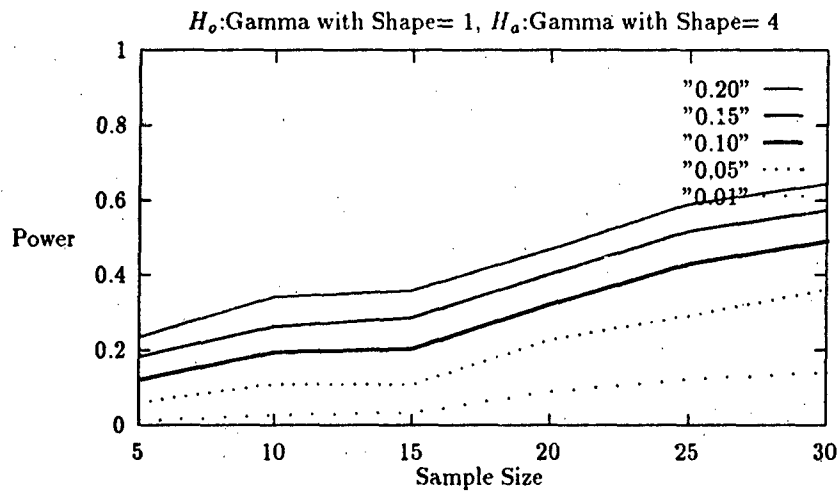


Figure 4.13. Power Study, Gamma Shape=1, Gamma Shape=4

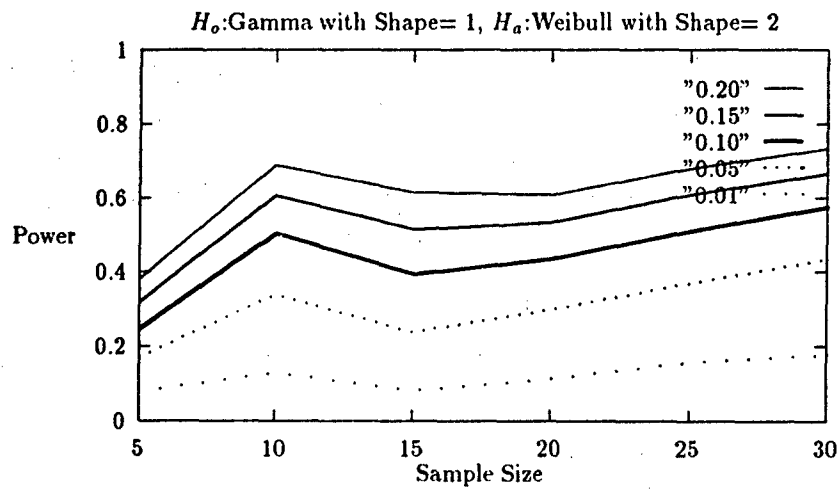


Figure 4.14. Power Study, Gamma Shape=1, Weibull Shape=2

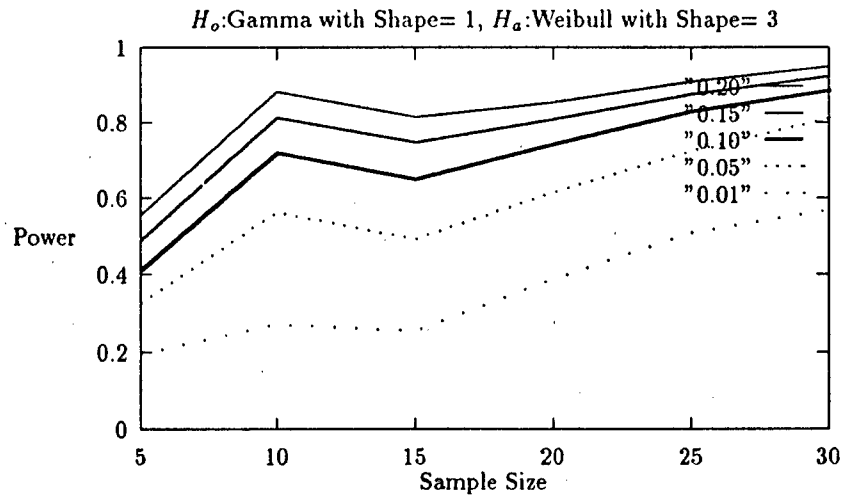


Figure 4.15. Power Study, Gamma Shape=1, Weibull Shape=3

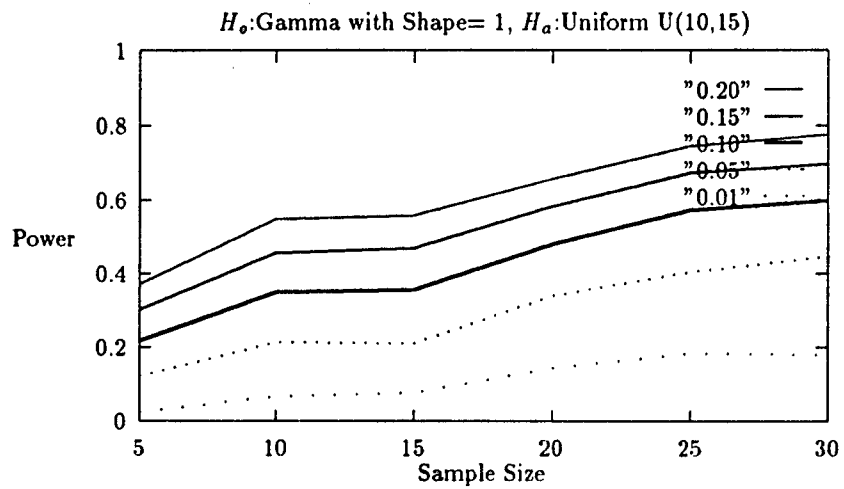


Figure 4.16. Power Study, Gamma Shape=1, Uniform U(10,15)

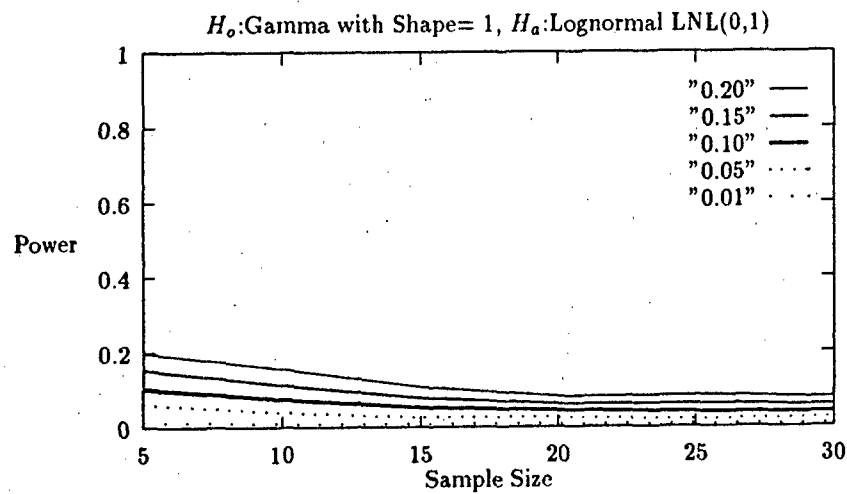


Figure 4.17. Power Study, Gamma Shape=1, Lognormal LNL(0,1)

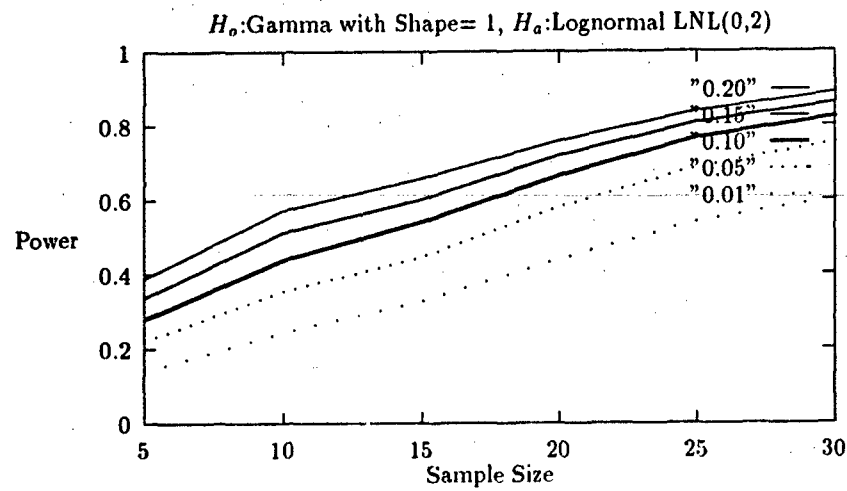


Figure 4.18. Power Study, Gamma Shape=1, Lognormal LNL(0,2)

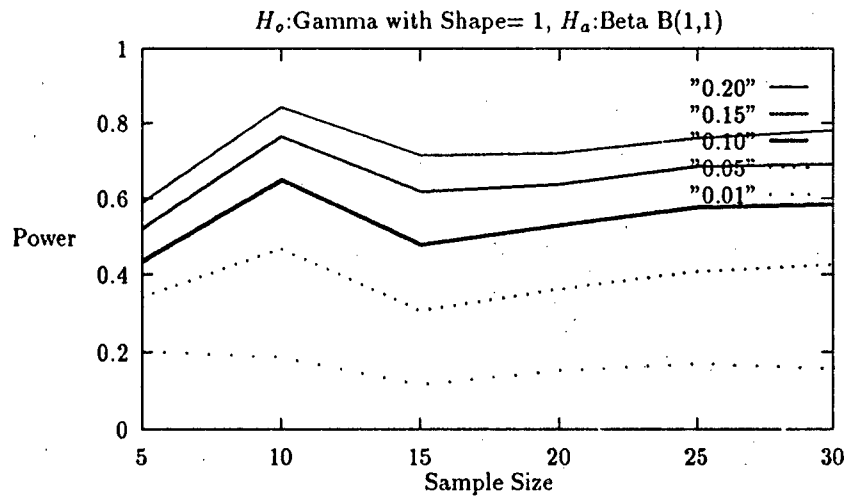


Figure 4.19. Power Study, Gamma Shape=1, Beta B(1,1)

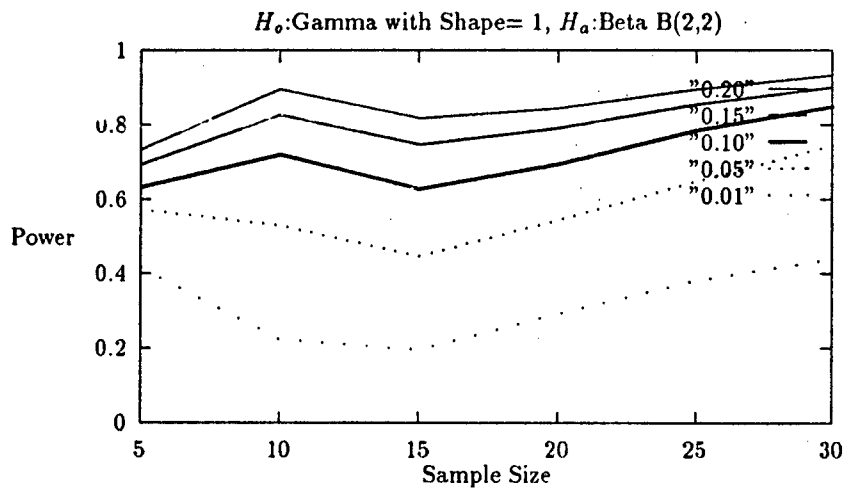


Figure 4.20. Power Study, Gamma Shape=1, Beta B(2,2)

H_0 : Gamma with Shape = 1.5; H_a : Another Distribution

| Sample Size | 1-x | Gamma Shape=1.5 | Gamma Shape=2.5 | Gamma Shape=4.0 | Weibull Shape=2.0 | Weibull Shape=3.0 |
|-------------|-----|-----------------|-----------------|-----------------|-------------------|-------------------|
| 5 | .20 | 0.19992 | 0.20412 | 0.19712 | 0.27049 | 0.34546 |
| | .15 | 0.14994 | 0.15754 | 0.14874 | 0.20792 | 0.27229 |
| | .10 | 0.09996 | 0.10696 | 0.10236 | 0.14654 | 0.19632 |
| | .05 | 0.04998 | 0.05138 | 0.04878 | 0.07457 | 0.10896 |
| | .01 | 0.01000 | 0.00900 | 0.01040 | 0.01399 | 0.02479 |
| 10 | .20 | 0.20692 | 0.22731 | 0.27549 | 0.36485 | 0.57357 |
| | .15 | 0.15394 | 0.17213 | 0.21811 | 0.28888 | 0.49980 |
| | .10 | 0.10096 | 0.11875 | 0.16074 | 0.21611 | 0.41144 |
| | .05 | 0.05118 | 0.06097 | 0.09756 | 0.13835 | 0.29768 |
| | .01 | 0.01000 | 0.01359 | 0.02219 | 0.03559 | 0.11515 |
| 15 | .20 | 0.19552 | 0.24110 | 0.35206 | 0.48241 | 0.77029 |
| | .15 | 0.14434 | 0.19012 | 0.28749 | 0.41184 | 0.72931 |
| | .10 | 0.10116 | 0.13475 | 0.22751 | 0.33966 | 0.67433 |
| | .05 | 0.04438 | 0.06397 | 0.12555 | 0.23011 | 0.58237 |
| | .01 | 0.01000 | 0.01140 | 0.03459 | 0.09956 | 0.34586 |
| 20 | .20 | 0.19352 | 0.25550 | 0.44462 | 0.63115 | 0.90524 |
| | .15 | 0.14054 | 0.19492 | 0.38145 | 0.57617 | 0.87445 |
| | .10 | 0.09396 | 0.13735 | 0.30328 | 0.50160 | 0.82907 |
| | .05 | 0.05418 | 0.08577 | 0.21012 | 0.40004 | 0.75210 |
| | .01 | 0.00860 | 0.01719 | 0.05838 | 0.15694 | 0.47441 |
| 25 | .20 | 0.21112 | 0.29188 | 0.49480 | 0.73431 | 0.95222 |
| | .15 | 0.15714 | 0.23131 | 0.42043 | 0.67213 | 0.93183 |
| | .10 | 0.10076 | 0.16995 | 0.34266 | 0.59056 | 0.89364 |
| | .05 | 0.04878 | 0.09656 | 0.22891 | 0.42883 | 0.80288 |
| | .01 | 0.01020 | 0.01819 | 0.06817 | 0.15414 | 0.51260 |
| 30 | .20 | 0.18813 | 0.29688 | 0.54058 | 0.77829 | 0.96621 |
| | .15 | 0.14474 | 0.24070 | 0.48001 | 0.71471 | 0.95202 |
| | .10 | 0.09496 | 0.17633 | 0.38904 | 0.62035 | 0.91783 |
| | .05 | 0.04638 | 0.10356 | 0.26350 | 0.45382 | 0.84526 |
| | .01 | 0.00980 | 0.02859 | 0.10656 | 0.18972 | 0.61016 |

Table 4.13. Power Test for Gamma, Shape = 1.5

H_0 : Gamma with Shape = 1.5; H_a : Another Distribution

| Sample Size | 1-x | Uniform (10,15) | Lognorm. $\omega = 0, \rho = 1$ | Lognorm. $\omega = 0, \rho = 2$ | Beta $p=1, q=1$ | Beta $p=2, q=2$ |
|-------------|-----|-----------------|---------------------------------|---------------------------------|-----------------|-----------------|
| 5 | .20 | 0.31867 | 0.61795 | 0.76789 | 0.88525 | 0.90864 |
| | .15 | 0.24850 | 0.52159 | 0.69672 | 0.81687 | 0.84666 |
| | .10 | 0.17633 | 0.39864 | 0.61735 | 0.70532 | 0.74950 |
| | .05 | 0.09736 | 0.23791 | 0.49660 | 0.52539 | 0.57937 |
| | .01 | 0.01839 | 0.06477 | 0.30988 | 0.21012 | 0.26789 |
| 10 | .20 | 0.58497 | 0.43343 | 0.83827 | 0.87245 | 0.92143 |
| | .15 | 0.49920 | 0.34646 | 0.79548 | 0.80968 | 0.88305 |
| | .10 | 0.41064 | 0.26270 | 0.74370 | 0.73311 | 0.83767 |
| | .05 | 0.28509 | 0.16354 | 0.65974 | 0.60156 | 0.76010 |
| | .01 | 0.09716 | 0.05818 | 0.48401 | 0.35206 | 0.56897 |
| 15 | .20 | 0.82807 | 0.42963 | 0.92983 | 0.91883 | 0.96561 |
| | .15 | 0.76350 | 0.36246 | 0.91224 | 0.88705 | 0.94562 |
| | .10 | 0.68093 | 0.29248 | 0.88105 | 0.83627 | 0.92143 |
| | .05 | 0.49720 | 0.19332 | 0.82107 | 0.70932 | 0.83107 |
| | .01 | 0.20652 | 0.07957 | 0.67293 | 0.38065 | 0.52919 |
| 20 | .20 | 0.87985 | 0.46941 | 0.96881 | 0.95322 | 0.98181 |
| | .15 | 0.82087 | 0.40104 | 0.95842 | 0.92423 | 0.96941 |
| | .10 | 0.73071 | 0.32727 | 0.94222 | 0.86925 | 0.94002 |
| | .05 | 0.59436 | 0.24930 | 0.91364 | 0.76449 | 0.86645 |
| | .01 | 0.24670 | 0.10236 | 0.80308 | 0.39944 | 0.54498 |
| 25 | .20 | 0.92363 | 0.50220 | 0.98900 | 0.97681 | 0.99080 |
| | .15 | 0.87205 | 0.43663 | 0.98281 | 0.95502 | 0.98261 |
| | .10 | 0.80168 | 0.36326 | 0.97601 | 0.90784 | 0.95362 |
| | .05 | 0.63795 | 0.24950 | 0.95702 | 0.76629 | 0.87205 |
| | .01 | 0.26310 | 0.09856 | 0.87705 | 0.35906 | 0.54098 |
| 30 | .20 | 0.93703 | 0.52339 | 0.99580 | 0.97581 | 0.99220 |
| | .15 | 0.90104 | 0.45682 | 0.99320 | 0.95662 | 0.98321 |
| | .10 | 0.83227 | 0.37745 | 0.98900 | 0.90884 | 0.96082 |
| | .05 | 0.67993 | 0.26709 | 0.97641 | 0.77949 | 0.89044 |
| | .01 | 0.34066 | 0.12475 | 0.92983 | 0.40784 | 0.62815 |

Table 4.14. Power Test for Gamma, Shape = 1.5 (Continue)

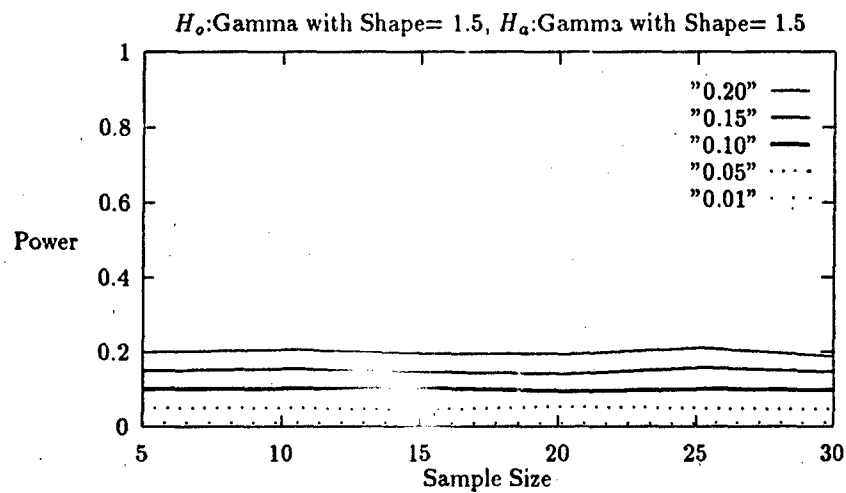


Figure 4.21. Power Study, Gamma Shape=1.5, Gamma Shape=1.5

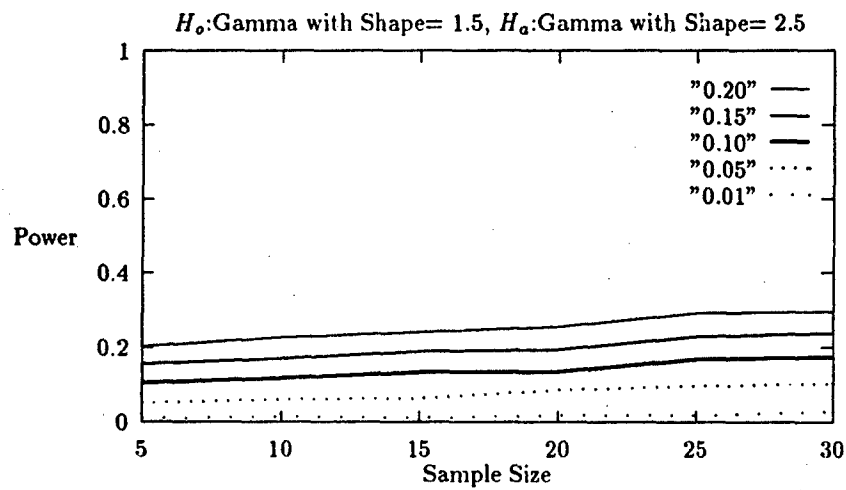


Figure 4.22. Power Study, Gamma Shape=1.5, Gamma Shape=2.5

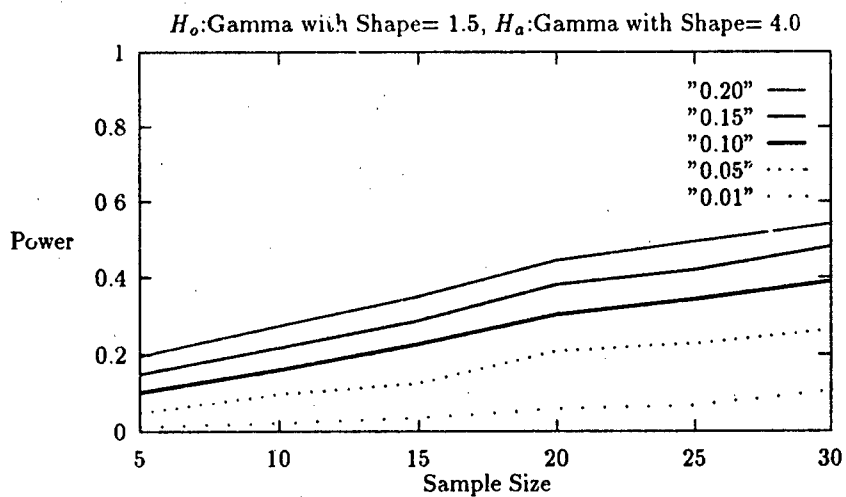


Figure 4.23. Power Study, Gamma Shape=1.5, Gamma Shape=4

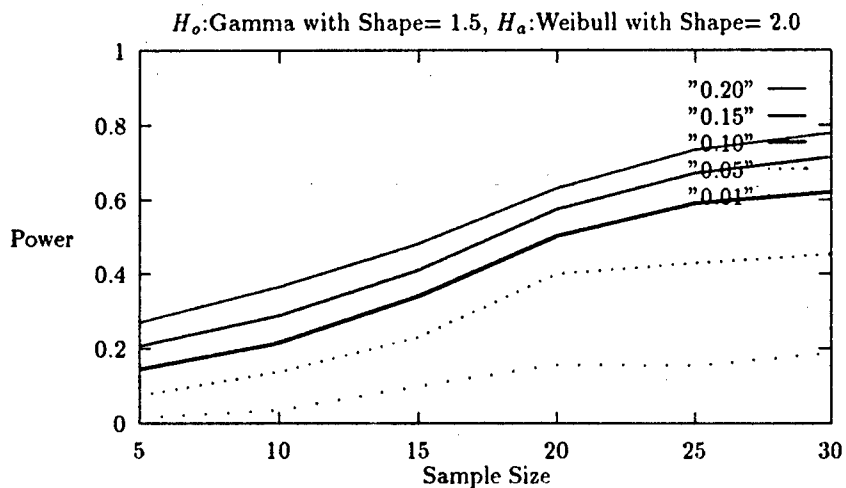


Figure 4.24. Power Study, Gamma Shape=1.5, Weibull Shape=2

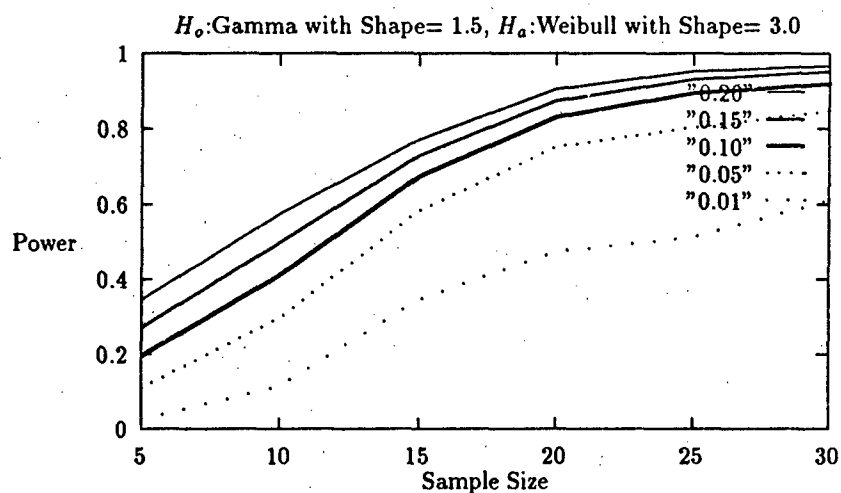


Figure 4.25. Power: Study, Gamma Shape=1.5, Weibull Shape=3

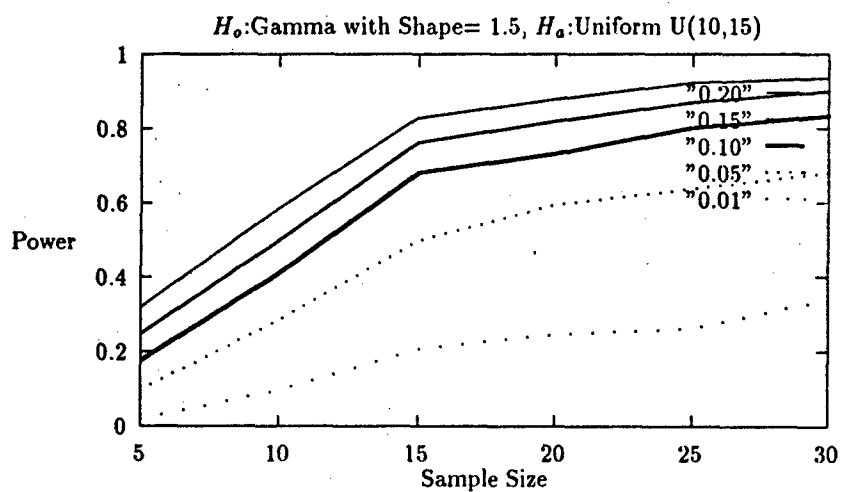


Figure 4.26. Power Study, Gamma Shape=1.5, Uniform U(10,15)

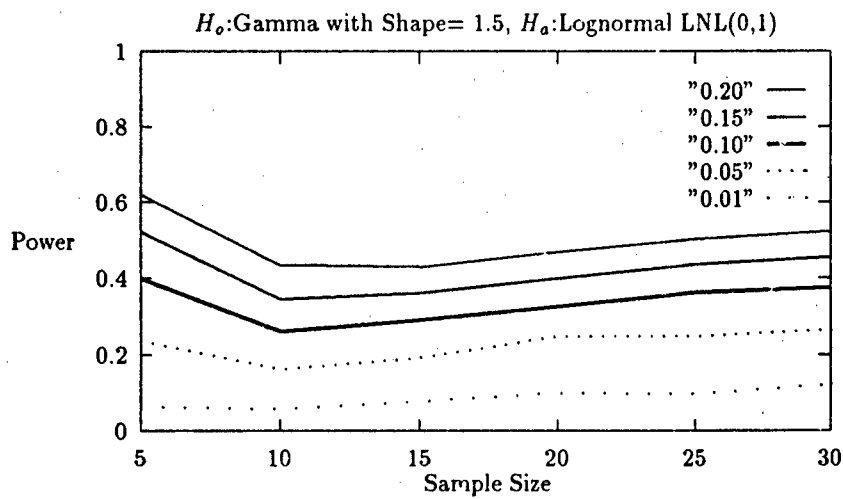


Figure 4.27. Power Study, Gamma Shape=1.5, Lognormal LNL(0,1)

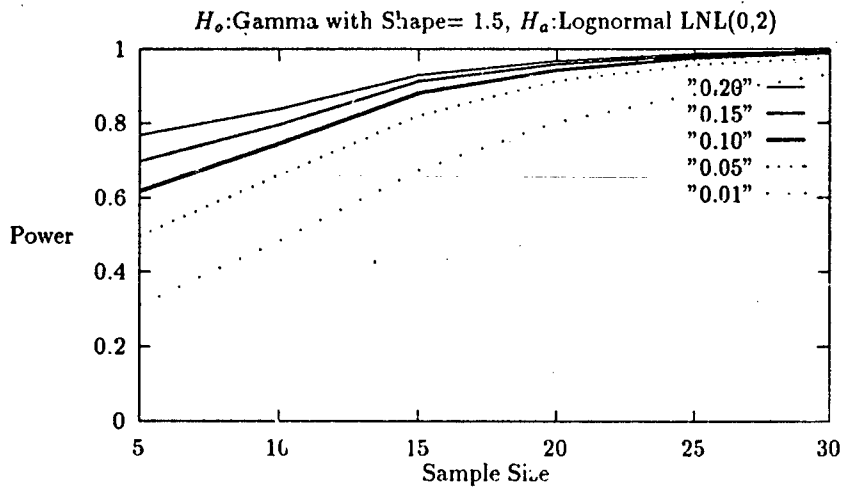


Figure 4.28. Power Study, Gamma Shape=1.5, Lognormal LNL(0,2)

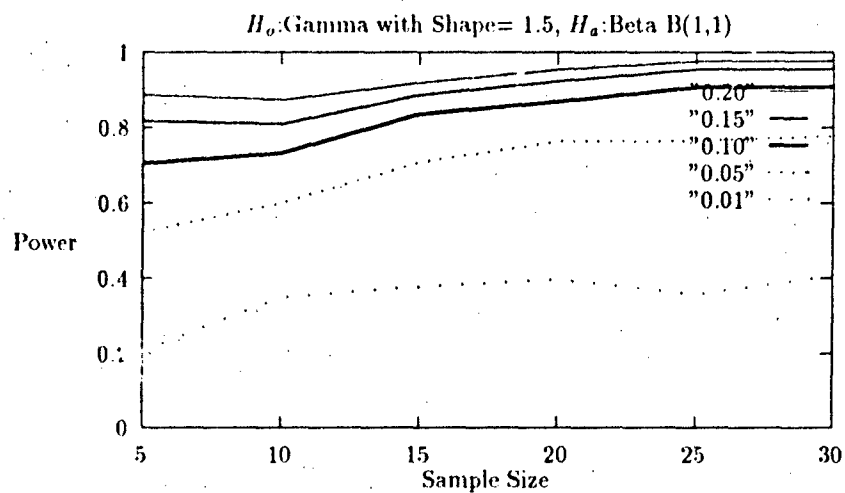


Figure 4.29. Power Study, Gamma Shape=1.5, Beta B(1,1)

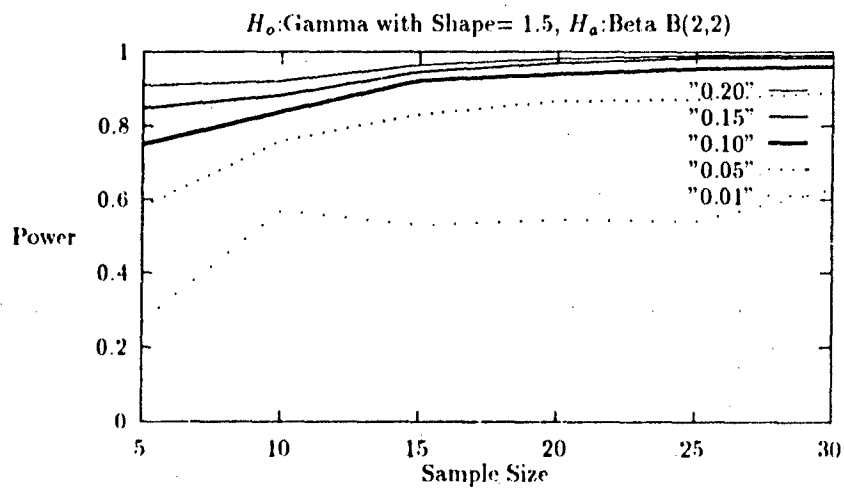


Figure 4.30. Power Study, Gamma Shape=1.5, Beta B(2,2)

H_0 : Gamma with Shape = 2.0; H_a : Another Distribution

| Sample Size | 1-x | Gamma Shape=2.0 | Gamma Shape=2.5 | Gamma Shape=4.0 | Weibull Shape=2.0 | Weibull Shape=3.0 |
|-------------|-----|-----------------|-----------------|-----------------|-------------------|-------------------|
| 5 | .20 | 0.19992 | 0.18353 | 0.17873 | 0.22231 | 0.28769 |
| | .15 | 0.14994 | 0.13155 | 0.13575 | 0.17113 | 0.23551 |
| | .10 | 0.09996 | 0.09056 | 0.09216 | 0.11995 | 0.17153 |
| | .05 | 0.04998 | 0.04558 | 0.04518 | 0.05758 | 0.08916 |
| | .01 | 0.01000 | 0.01040 | 0.01240 | 0.01120 | 0.02519 |
| 10 | .20 | 0.19672 | 0.20232 | 0.22631 | 0.29288 | 0.45642 |
| | .15 | 0.15054 | 0.15074 | 0.17853 | 0.22831 | 0.38705 |
| | .10 | 0.09576 | 0.10116 | 0.12315 | 0.15934 | 0.31188 |
| | .05 | 0.05018 | 0.05538 | 0.07357 | 0.09156 | 0.22251 |
| | .01 | 0.00980 | 0.01140 | 0.01919 | 0.02019 | 0.07957 |
| 15 | .20 | 0.20132 | 0.20392 | 0.26419 | 0.35226 | 0.61316 |
| | .15 | 0.15394 | 0.15594 | 0.22171 | 0.29608 | 0.55178 |
| | .10 | 0.10376 | 0.10236 | 0.16094 | 0.22671 | 0.47821 |
| | .05 | 0.05458 | 0.05258 | 0.09856 | 0.14374 | 0.36026 |
| | .01 | 0.01159 | 0.01100 | 0.02779 | 0.04778 | 0.18533 |
| 20 | .20 | 0.19172 | 0.20752 | 0.32007 | 0.42103 | 0.74170 |
| | .15 | 0.14954 | 0.15894 | 0.26869 | 0.36545 | 0.69512 |
| | .10 | 0.09716 | 0.11615 | 0.20612 | 0.28569 | 0.63275 |
| | .05 | 0.05538 | 0.06118 | 0.14194 | 0.19912 | 0.53739 |
| | .01 | 0.01159 | 0.01559 | 0.04338 | 0.07677 | 0.35486 |
| 25 | .20 | 0.20092 | 0.21591 | 0.34466 | 0.48741 | 0.83647 |
| | .15 | 0.15734 | 0.17413 | 0.29028 | 0.42563 | 0.80968 |
| | .10 | 0.10396 | 0.11875 | 0.22671 | 0.34346 | 0.76150 |
| | .05 | 0.05198 | 0.06178 | 0.14154 | 0.24650 | 0.68333 |
| | .01 | 0.00940 | 0.01659 | 0.05158 | 0.12975 | 0.49200 |
| 30 | .20 | 0.18393 | 0.21172 | 0.39304 | 0.54698 | 0.90084 |
| | .15 | 0.13994 | 0.16753 | 0.33607 | 0.49540 | 0.87985 |
| | .10 | 0.09656 | 0.11755 | 0.27009 | 0.43183 | 0.85286 |
| | .05 | 0.04838 | 0.06717 | 0.18253 | 0.34066 | 0.79068 |
| | .01 | 0.01019 | 0.01260 | 0.07017 | 0.16493 | 0.59976 |

Table 4.15. Power Test for Gamma, Shape = 2.0

H_0 : Gamma with Shape = 2.0; H_a : Another Distribution

| Sample Size | 1-x | Uniform (10,15) | Lognorm. $\omega = 0, \rho = 1$ | Lognorm. $\omega = 0, \rho = 2$ | Beta $p=1, q=1$ | Beta $p=2, q=2$ |
|-------------|-----|-----------------|---------------------------------|---------------------------------|-----------------|-----------------|
| 5 | .20 | 0.28409 | 0.78888 | 0.88825 | 0.95542 | 0.97481 |
| | .15 | 0.22051 | 0.68273 | 0.82627 | 0.91583 | 0.93623 |
| | .10 | 0.16134 | 0.56677 | 0.75470 | 0.84206 | 0.86645 |
| | .05 | 0.08617 | 0.37665 | 0.62555 | 0.67473 | 0.72011 |
| | .01 | 0.01759 | 0.13475 | 0.42243 | 0.32107 | 0.35106 |
| 10 | .20 | 0.60416 | 0.55418 | 0.90144 | 0.89284 | 0.90924 |
| | .15 | 0.53139 | 0.45822 | 0.86945 | 0.84166 | 0.85946 |
| | .10 | 0.43423 | 0.35546 | 0.82527 | 0.74890 | 0.78489 |
| | .05 | 0.31048 | 0.23651 | 0.76549 | 0.58896 | 0.64694 |
| | .01 | 0.11236 | 0.08497 | 0.61715 | 0.27649 | 0.39284 |
| 15 | .20 | 0.85306 | 0.56497 | 0.95602 | 0.90804 | 0.93583 |
| | .15 | 0.79708 | 0.50420 | 0.94502 | 0.86286 | 0.91064 |
| | .10 | 0.70152 | 0.41743 | 0.92463 | 0.78849 | 0.86745 |
| | .05 | 0.53819 | 0.31128 | 0.88545 | 0.66074 | 0.78149 |
| | .01 | 0.26210 | 0.16034 | 0.79148 | 0.42543 | 0.60476 |
| 20 | .20 | 0.89324 | 0.61995 | 0.98781 | 0.94602 | 0.96042 |
| | .15 | 0.84366 | 0.56038 | 0.98281 | 0.91883 | 0.94502 |
| | .10 | 0.76589 | 0.47821 | 0.97361 | 0.86565 | 0.90984 |
| | .05 | 0.63435 | 0.38545 | 0.95762 | 0.76070 | 0.84966 |
| | .01 | 0.32627 | 0.21411 | 0.90144 | 0.46861 | 0.61875 |
| 25 | .20 | 0.91863 | 0.68513 | 0.99540 | 0.95422 | 0.97621 |
| | .15 | 0.88785 | 0.63855 | 0.99400 | 0.93283 | 0.96481 |
| | .10 | 0.81987 | 0.56957 | 0.99240 | 0.88685 | 0.94362 |
| | .05 | 0.67993 | 0.44942 | 0.98621 | 0.77849 | 0.87885 |
| | .01 | 0.37465 | 0.26509 | 0.95842 | 0.48980 | 0.65734 |
| 30 | .20 | 0.95302 | 0.73251 | 0.99900 | 0.97381 | 0.98481 |
| | .15 | 0.92183 | 0.68713 | 0.99860 | 0.95802 | 0.97721 |
| | .10 | 0.87185 | 0.62635 | 0.99780 | 0.92523 | 0.95702 |
| | .05 | 0.77069 | 0.52639 | 0.99580 | 0.85006 | 0.90764 |
| | .01 | 0.45782 | 0.32847 | 0.98521 | 0.57777 | 0.70752 |

Table 4.16. Power Test for Gamma, Shape = 2.0 (Continue)

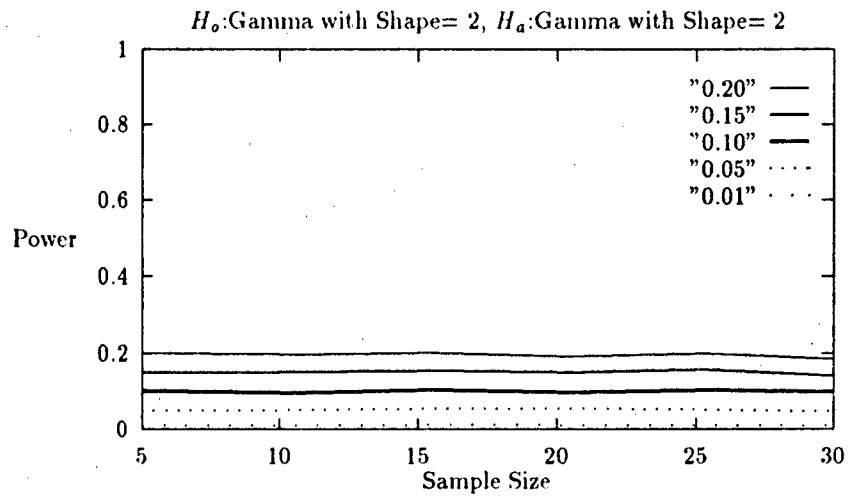


Figure 4.31. Power Study, Gamma Shape=2, Gamma Shape=2

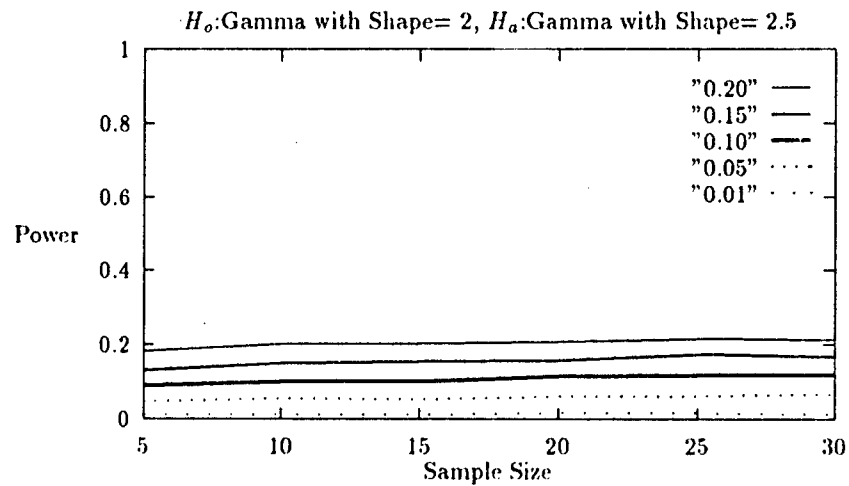


Figure 4.32. Power Study, Gamma Shape=2, Gamma Shape=2.5

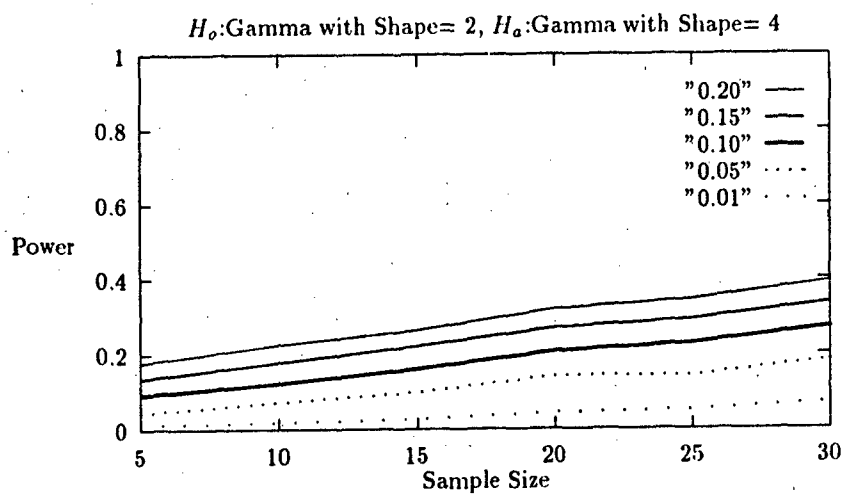


Figure 4.33. Power Study, Gamma Shape=2, Gamma Shape=4

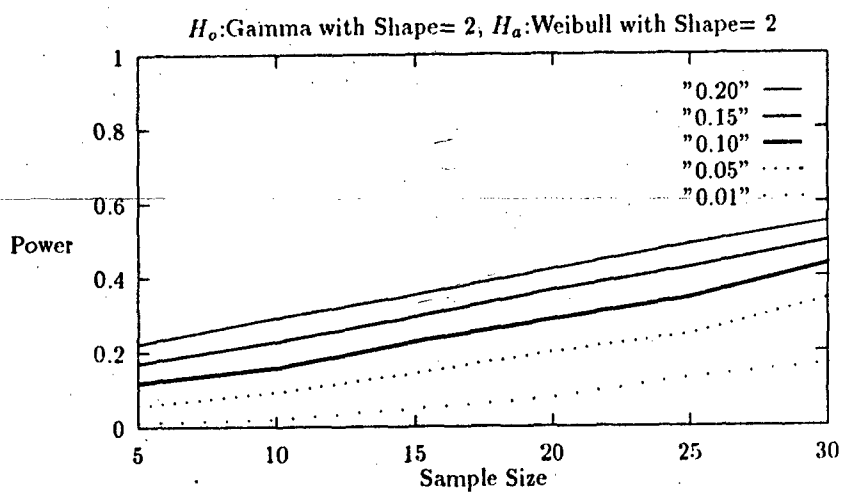


Figure 4.34. Power Study, Gamma Shape=2, Weibull Shape=2

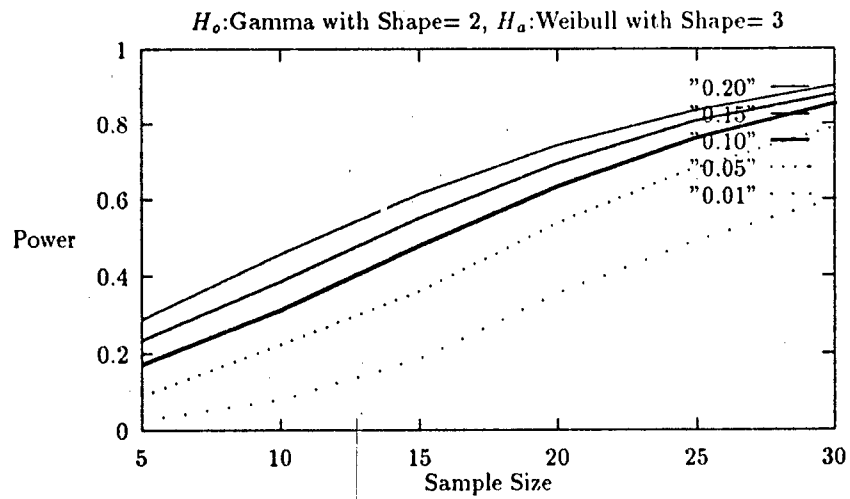


Figure 4.35. Power Study, Gamma Shape=2, Weibull Shape=3

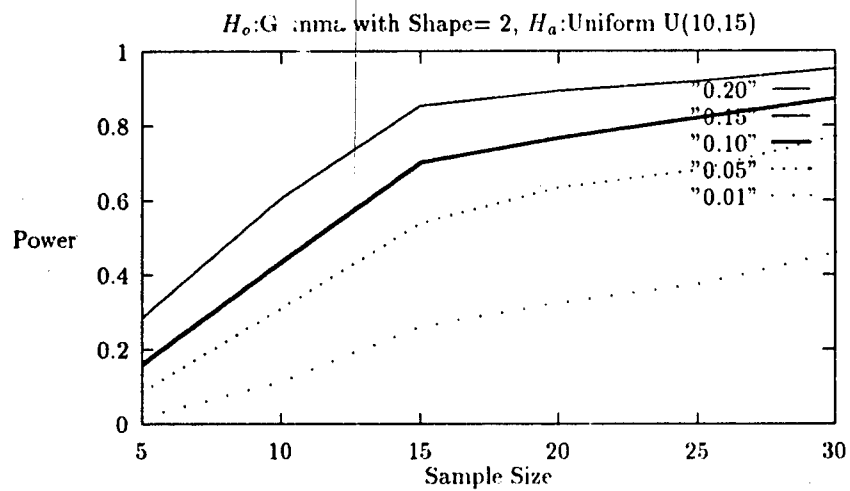


Figure 4.36. Power Study, Gamma Shape=2, Uniform U(10,15)

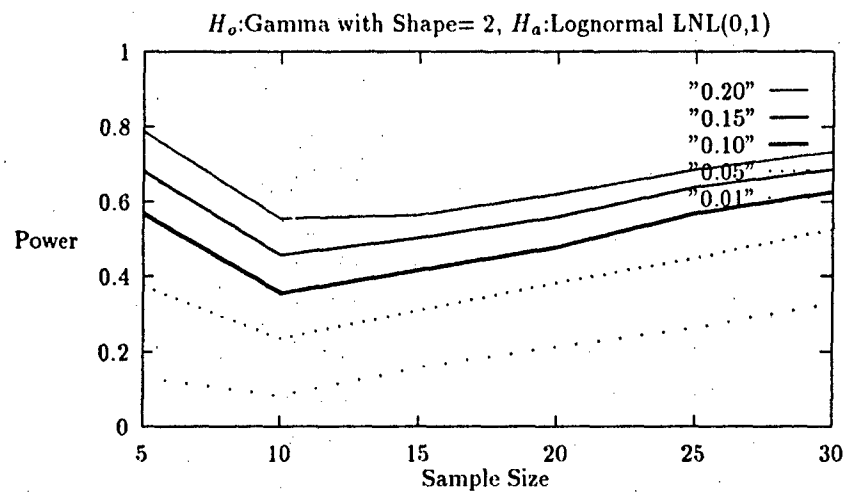


Figure 4.37. Power Study, Gamma Shape=2, Lognormal LNL(0,1)

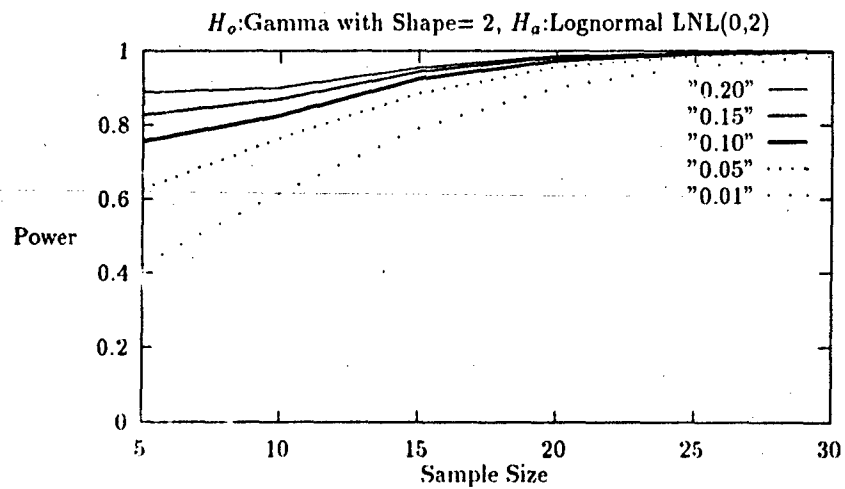


Figure 4.38. Power Study, Gamma Shape=2, Lognormal LNL(0,2)

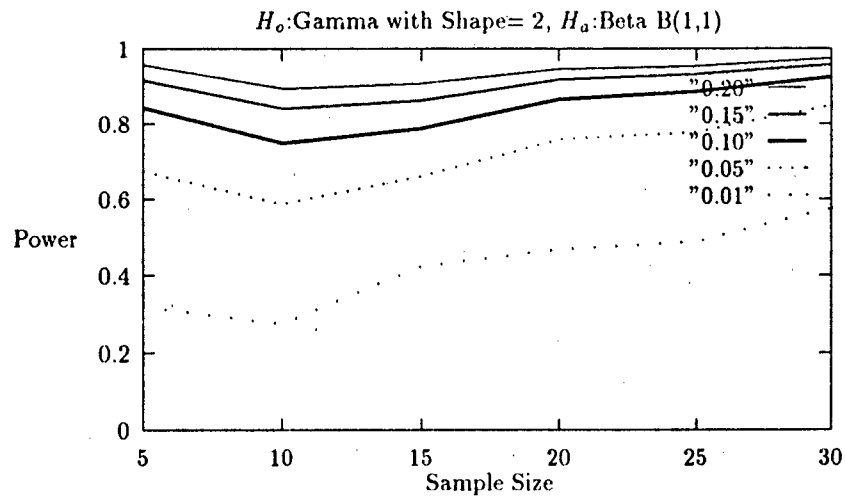


Figure 4.39. Power Study, Gamma Shape=2, Beta B(1,1)

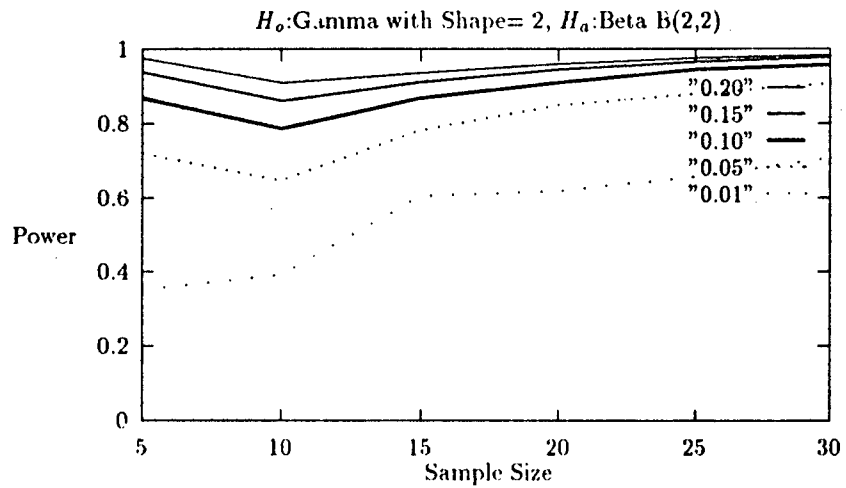


Figure 4.40. Power Study, Gamma Shape=2, Beta B(2,2)

H_0 : Gamma with Shape = 3.0; H_a : Another Distribution

| Sample Size | 1-x | Gamma Shape=3.0 | Gamma Shape=2.5 | Gamma Shape=4.0 | Weibull Shape=2.0 | Weibull Shape=3.0 |
|-------------|-----|-----------------|-----------------|-----------------|-------------------|-------------------|
| 5 | .20 | 0.19992 | 0.20932 | 0.20872 | 0.21531 | 0.26370 |
| | .15 | 0.14994 | 0.15454 | 0.16334 | 0.16114 | 0.20152 |
| | .10 | 0.09996 | 0.09796 | 0.11036 | 0.10276 | 0.14354 |
| | .05 | 0.04998 | 0.05278 | 0.05578 | 0.04858 | 0.07277 |
| | .01 | 0.01000 | 0.00920 | 0.01120 | 0.00820 | 0.01339 |
| 10 | .20 | 0.20692 | 0.19572 | 0.18892 | 0.23111 | 0.34706 |
| | .15 | 0.15314 | 0.14674 | 0.13755 | 0.17213 | 0.28069 |
| | .10 | 0.10216 | 0.09696 | 0.09336 | 0.12375 | 0.20992 |
| | .05 | 0.04978 | 0.04778 | 0.04758 | 0.06577 | 0.13055 |
| | .01 | 0.00860 | 0.01060 | 0.01120 | 0.01839 | 0.04038 |
| 15 | .20 | 0.19772 | 0.18053 | 0.19612 | 0.23291 | 0.43843 |
| | .15 | 0.15914 | 0.14154 | 0.14854 | 0.19112 | 0.37865 |
| | .10 | 0.10676 | 0.08916 | 0.09996 | 0.13335 | 0.30268 |
| | .05 | 0.05058 | 0.03978 | 0.04858 | 0.06857 | 0.19472 |
| | .01 | 0.01259 | 0.00700 | 0.01180 | 0.01759 | 0.06617 |
| 20 | .20 | 0.20832 | 0.19292 | 0.20092 | 0.25530 | 0.53079 |
| | .15 | 0.15574 | 0.13894 | 0.15174 | 0.19312 | 0.45562 |
| | .10 | 0.10196 | 0.09216 | 0.09776 | 0.13375 | 0.37005 |
| | .05 | 0.05578 | 0.04418 | 0.05318 | 0.07317 | 0.25850 |
| | .01 | 0.01119 | 0.00760 | 0.01060 | 0.01619 | 0.10516 |
| 25 | .20 | 0.20292 | 0.16973 | 0.18892 | 0.27129 | 0.59476 |
| | .15 | 0.14794 | 0.11795 | 0.14034 | 0.20732 | 0.52439 |
| | .10 | 0.10756 | 0.07777 | 0.09856 | 0.15054 | 0.45002 |
| | .05 | 0.05358 | 0.04198 | 0.04978 | 0.08057 | 0.33627 |
| | .01 | 0.01119 | 0.00660 | 0.00780 | 0.01839 | 0.13415 |
| 30 | .20 | 0.20232 | 0.16294 | 0.18573 | 0.26509 | 0.64654 |
| | .15 | 0.15194 | 0.11575 | 0.13954 | 0.20692 | 0.58757 |
| | .10 | 0.10576 | 0.07277 | 0.09536 | 0.15014 | 0.51379 |
| | .05 | 0.05418 | 0.03519 | 0.05438 | 0.08597 | 0.39824 |
| | .01 | 0.00980 | 0.00520 | 0.01319 | 0.01859 | 0.17813 |

Table 4.17. Power Test for Gamma, Shape = 3.0

H_0 : Gamma with Shape = 3.0; H_a : Another Distribution

| Sample Size | 1-x | Uniform (10,15) | Lognorm. $\omega = 0, \rho = 1$ | Lognorm. $\omega = 0, \rho = 2$ | Beta $p=1, q=1$ | Beta $p=2, q=2$ |
|-------------|-----|-----------------|---------------------------------|---------------------------------|-----------------|-----------------|
| 5 | .20 | 0.28089 | 0.99960 | 0.99960 | 0.99960 | 0.99960 |
| | .15 | 0.21252 | 0.98521 | 0.99200 | 0.99960 | 0.99940 |
| | .10 | 0.14354 | 0.90664 | 0.95342 | 0.98940 | 0.98960 |
| | .05 | 0.07357 | 0.72211 | 0.85046 | 0.93463 | 0.94102 |
| | .01 | 0.01279 | 0.31527 | 0.59616 | 0.64994 | 0.67073 |
| 10 | .20 | 0.39464 | 0.74650 | 0.95102 | 0.95942 | 0.96042 |
| | .15 | 0.31867 | 0.64994 | 0.92683 | 0.92563 | 0.92643 |
| | .10 | 0.23631 | 0.53119 | 0.89244 | 0.86126 | 0.87645 |
| | .05 | 0.14174 | 0.36366 | 0.84146 | 0.71991 | 0.74890 |
| | .01 | 0.03938 | 0.16393 | 0.73691 | 0.38805 | 0.42003 |
| 15 | .20 | 0.86186 | 0.69472 | 0.98241 | 0.93643 | 0.92763 |
| | .15 | 0.81667 | 0.63295 | 0.97481 | 0.90024 | 0.89464 |
| | .10 | 0.72651 | 0.54018 | 0.96302 | 0.82287 | 0.82327 |
| | .05 | 0.55258 | 0.39744 | 0.93703 | 0.65214 | 0.68033 |
| | .01 | 0.24870 | 0.20112 | 0.87005 | 0.31248 | 0.41823 |
| 20 | .20 | 0.89884 | 0.76489 | 0.99540 | 0.93623 | 0.93263 |
| | .15 | 0.84366 | 0.69832 | 0.99220 | 0.88465 | 0.89524 |
| | .10 | 0.75430 | 0.61336 | 0.98801 | 0.81368 | 0.82467 |
| | .05 | 0.60856 | 0.50680 | 0.97881 | 0.67513 | 0.70812 |
| | .01 | 0.27369 | 0.30068 | 0.94482 | 0.35886 | 0.46561 |
| 25 | .20 | 0.90464 | 0.79668 | 0.99860 | 0.93982 | 0.93043 |
| | .15 | 0.85466 | 0.74150 | 0.99840 | 0.89724 | 0.89124 |
| | .10 | 0.78189 | 0.67593 | 0.99600 | 0.83847 | 0.83607 |
| | .05 | 0.62135 | 0.57037 | 0.99280 | 0.70572 | 0.72191 |
| | .01 | 0.26569 | 0.35346 | 0.97521 | 0.36106 | 0.43942 |
| 30 | .20 | 0.92463 | 0.83847 | 0.99960 | 0.95042 | 0.94702 |
| | .15 | 0.88145 | 0.79728 | 0.99940 | 0.91483 | 0.91423 |
| | .10 | 0.81627 | 0.74010 | 0.99940 | 0.85746 | 0.85986 |
| | .05 | 0.68633 | 0.64974 | 0.99880 | 0.72731 | 0.75670 |
| | .01 | 0.33307 | 0.42863 | 0.99220 | 0.38325 | 0.44202 |

Table 4.18. Power Test for Gamma, Shape = 3.0 (Continue)

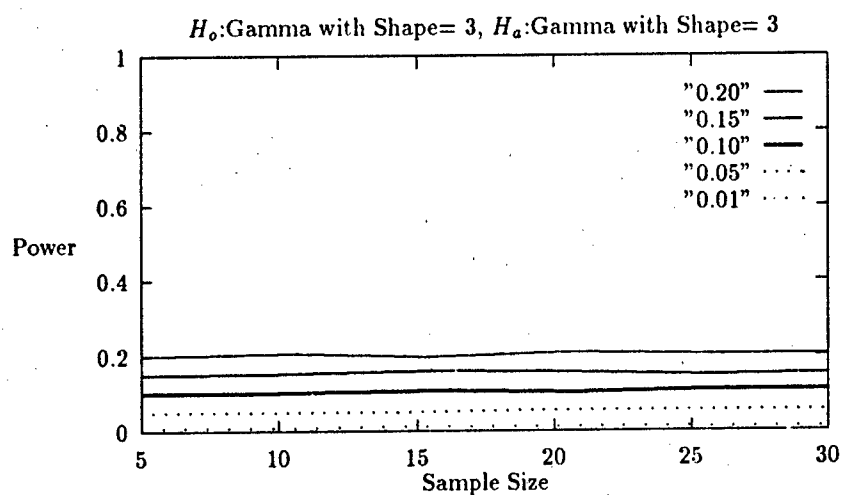


Figure 4.41. Power Study, Gamma Shape=3, Gamma Shape=3

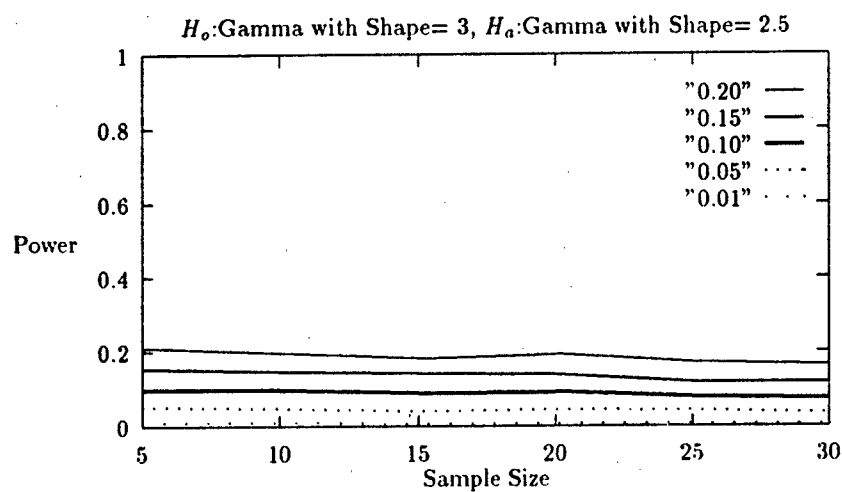


Figure 4.42. Power Study, Gamma Shape=3, Gamma Shape=2.5

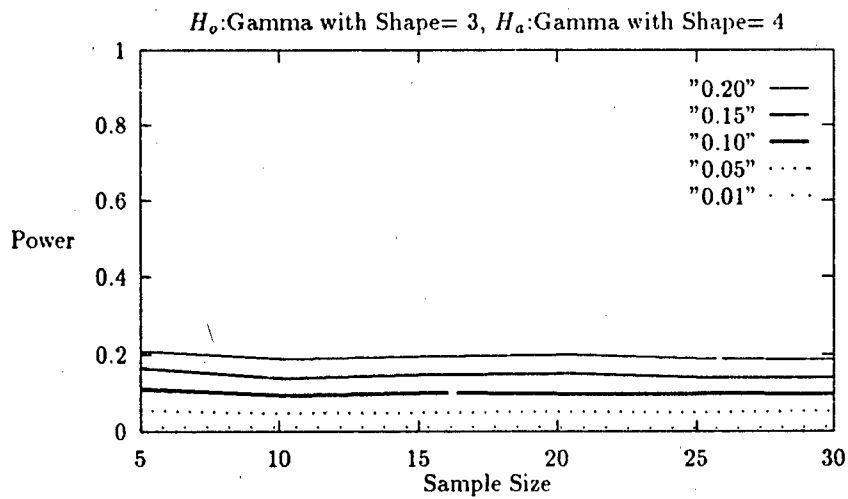


Figure 4.43. Power Study, Gamma Shape=3, Gamma Shape=4

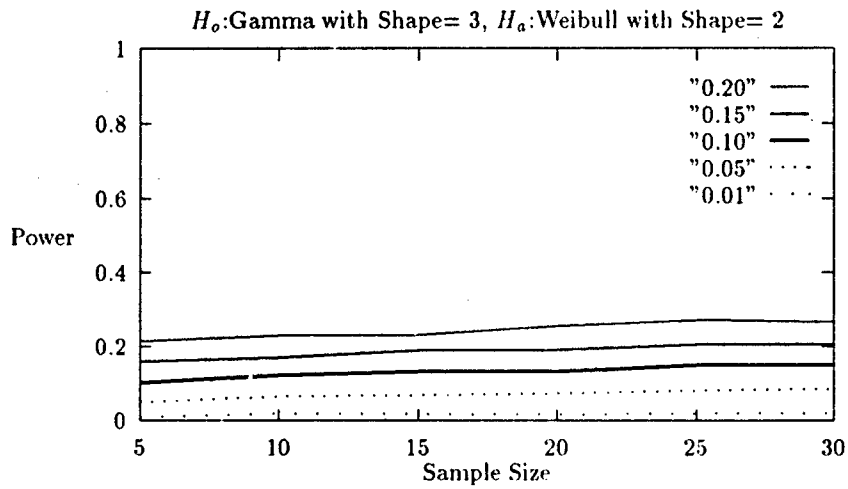


Figure 4.44. Power Study, Gamma Shape=3, Weibull Shape=2

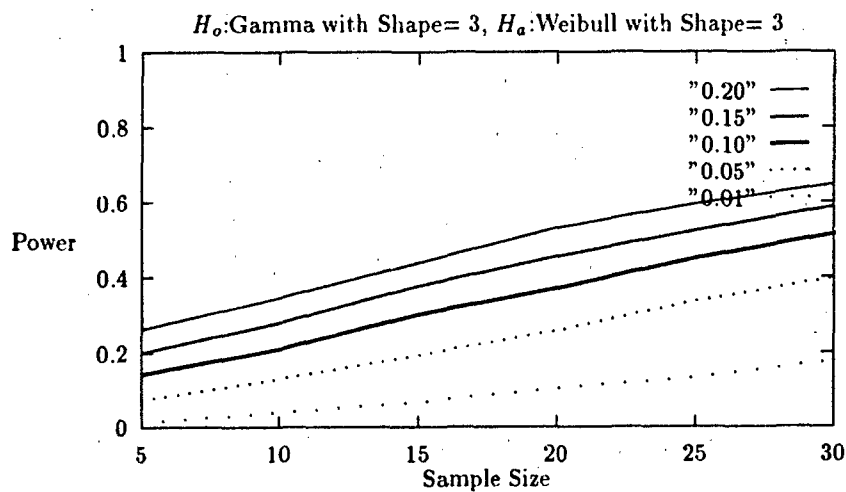


Figure 4.45. Power Study, Gamma Shape=3, Weibull Shape=3

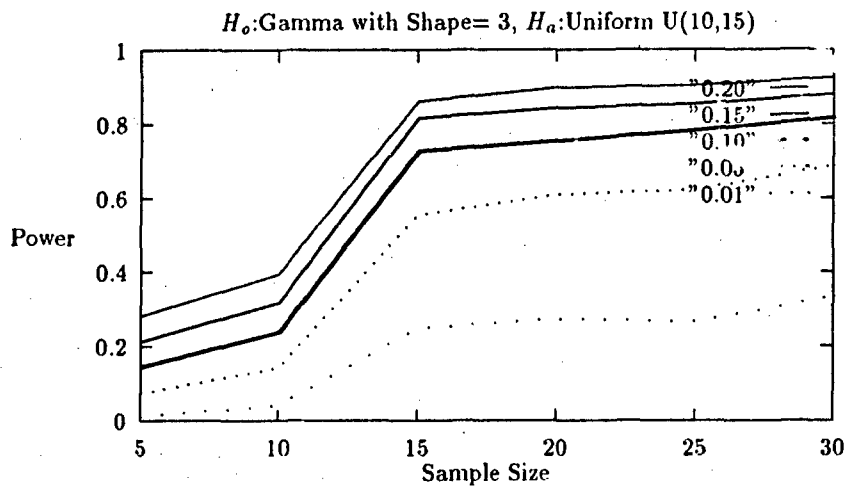


Figure 4.46. Power Study, Gamma Shape=3, Uniform U(10,15)

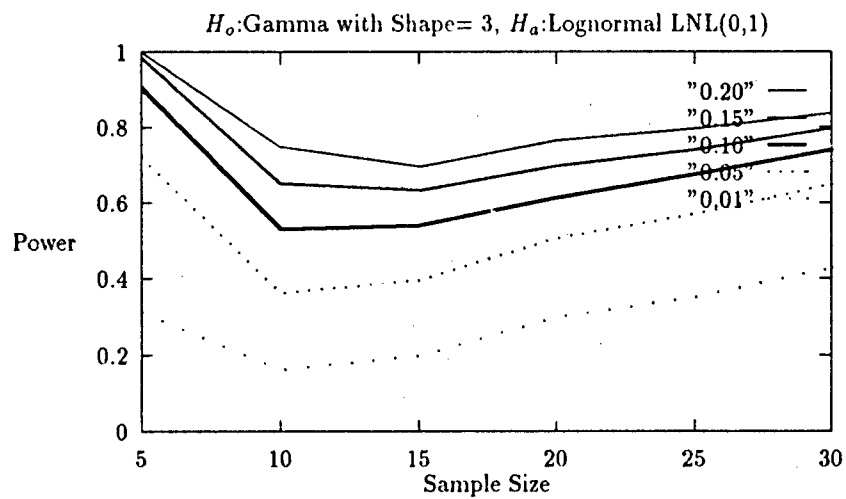


Figure 4.47. Power Study, Gamma Shape=3, Lognormal LNL(0,1)

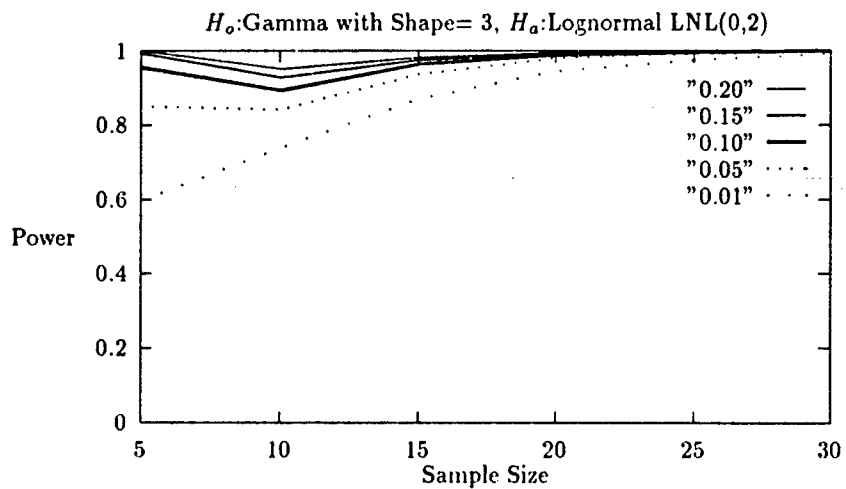


Figure 4.48. Power Study, Gamma Shape=3, Lognormal LNL(0,2)

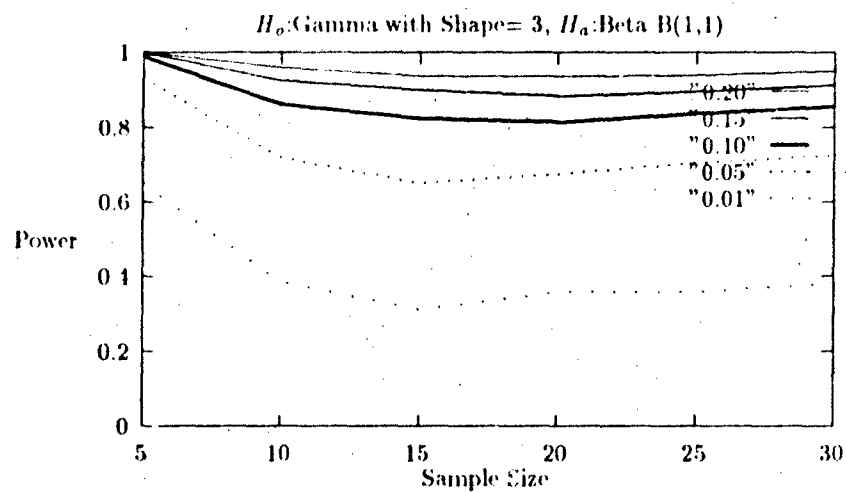


Figure 4.49. Power Study, Gamma Shape=3, Beta B(1,1)

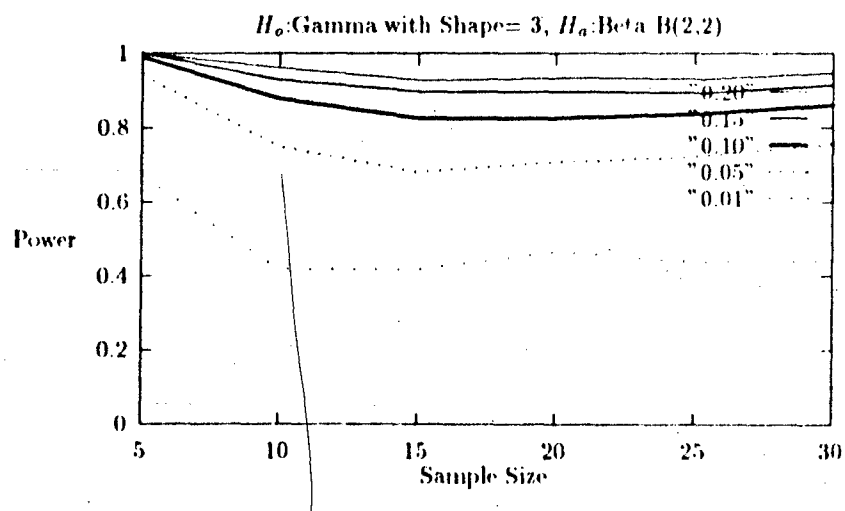


Figure 4.50. Power Study, Gamma Shape=3, Beta B(2,2)

H_0 :Gamma with Shape = 1.5; H_a :Another Distribution

| Sample Size \Rightarrow | | n=5 | | n=15 | | n=25 | |
|------------------------------------|------|-------|-------|-------|-------|-------|-------|
| Distribution | 1-x | Pre. | Mod. | Pre. | Mod. | Pre. | Mod. |
| Gamma Shape=1.5 | 0.01 | 0.011 | 0.010 | 0.009 | 0.010 | 0.011 | 0.010 |
| | 0.05 | 0.049 | 0.050 | 0.045 | 0.044 | 0.044 | 0.049 |
| Gamma Shape=2.5 | 0.01 | 0.006 | 0.009 | 0.004 | 0.011 | 0.015 | 0.018 |
| | 0.05 | 0.030 | 0.051 | 0.037 | 0.064 | 0.070 | 0.097 |
| Gamma Shape=4.0 | 0.01 | 0.005 | 0.010 | 0.011 | 0.035 | 0.064 | 0.068 |
| | 0.05 | 0.026 | 0.049 | 0.069 | 0.126 | 0.175 | 0.229 |
| Weibull Shape=2.0 | 0.01 | 0.002 | 0.014 | 0.020 | 0.100 | 0.102 | 0.154 |
| | 0.05 | 0.017 | 0.075 | 0.093 | 0.230 | 0.247 | 0.429 |
| Weibull Shape=3.0 | 0.01 | 0.001 | 0.025 | 0.086 | 0.346 | 0.403 | 0.513 |
| | 0.05 | 0.015 | 0.109 | 0.280 | 0.582 | 0.628 | 0.803 |
| Lognorm. $\omega = 0, \rho = 1$ | 0.01 | 0.047 | 0.065 | 0.167 | 0.080 | 0.293 | 0.099 |
| | 0.05 | 0.122 | 0.238 | 0.307 | 0.193 | 0.444 | 0.250 |
| Lognorm. $\omega = 0, \rho = 2$ | 0.01 | 0.260 | 0.310 | 0.844 | 0.673 | 0.981 | 0.877 |
| | 0.05 | 0.423 | 0.497 | 0.928 | 0.821 | 0.995 | 0.957 |
| Beta p=1,q=1 | 0.01 | 0.002 | 0.210 | 0.038 | 0.381 | 0.166 | 0.359 |
| | 0.05 | 0.020 | 0.525 | 0.184 | 0.709 | 0.413 | 0.766 |
| Beta p=2,q=2 | 0.01 | 0.001 | 0.268 | 0.066 | 0.529 | 0.302 | 0.541 |
| | 0.05 | 0.017 | 0.579 | 0.234 | 0.831 | 0.566 | 0.872 |

Table 4.19. The Comparison Table of the Modified and the Previous Power Studies, When Shape = 1.5

H_0 : Gamma with Shape = 4.0; H_a : Another Distribution

| Sample Size \Rightarrow | | n=5 | | n=15 | | n=25 | |
|------------------------------------|------|-------|-------|-------|-------|-------|-------|
| Distribution | 1-x | Pre. | Mod. | Pre. | Mod. | Pre. | Mod. |
| Gamma Shape=2.5 | 0.01 | 0.016 | 0.009 | 0.017 | 0.009 | 0.017 | 0.010 |
| | 0.05 | 0.063 | 0.046 | 0.064 | 0.050 | 0.075 | 0.041 |
| Gamma Shape=4.0 | 0.01 | 0.012 | 0.010 | 0.009 | 0.012 | 0.008 | 0.010 |
| | 0.05 | 0.054 | 0.050 | 0.045 | 0.055 | 0.048 | 0.049 |
| Weibull Shape=2.0 | 0.01 | 0.007 | 0.003 | 0.010 | 0.006 | 0.008 | 0.005 |
| | 0.05 | 0.042 | 0.037 | 0.050 | 0.042 | 0.054 | 0.037 |
| Weibull Shape=3.0 | 0.01 | 0.007 | 0.004 | 0.037 | 0.030 | 0.074 | 0.057 |
| | 0.05 | 0.040 | 0.048 | 0.119 | 0.116 | 0.218 | 0.178 |
| Lognorm. $\omega = 0, \rho = 1$ | 0.01 | 0.077 | 0.457 | 0.355 | 0.253 | 0.558 | 0.378 |
| | 0.05 | 0.175 | 0.917 | 0.517 | 0.495 | 0.761 | 0.601 |
| Lognorm. $\omega = 0, \rho = 2$ | 0.01 | 0.340 | 0.707 | 0.907 | 0.901 | 0.991 | 0.987 |
| | 0.05 | 0.489 | 0.962 | 0.963 | 0.956 | 0.999 | 0.995 |
| Beta p=1,q=1 | 0.01 | 0.008 | 0.801 | 0.037 | 0.350 | 0.075 | 0.303 |
| | 0.05 | 0.053 | 0.989 | 0.163 | 0.719 | 0.296 | 0.649 |
| Beta p=2,q=2 | 0.01 | 0.006 | 0.830 | 0.034 | 0.361 | 0.054 | 0.300 |
| | 0.05 | 0.044 | 0.995 | 0.117 | 0.712 | 0.204 | 0.594 |

Table 4.20. The Comparison Table of the Modified and the Previous Power Studies, When Shape = 4.0

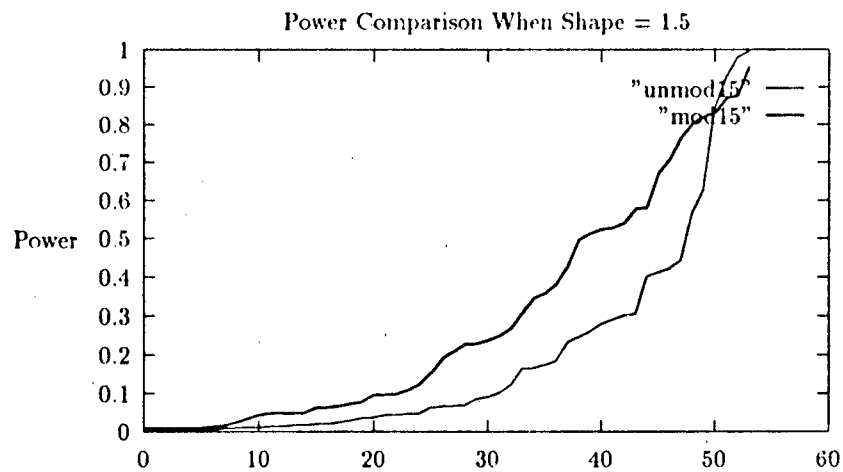


Figure 4.51. Graphical Comparison of the Previous and Modified Power Studies When Shape = 1.5 (Graph is prepared by taking the average of powers.)

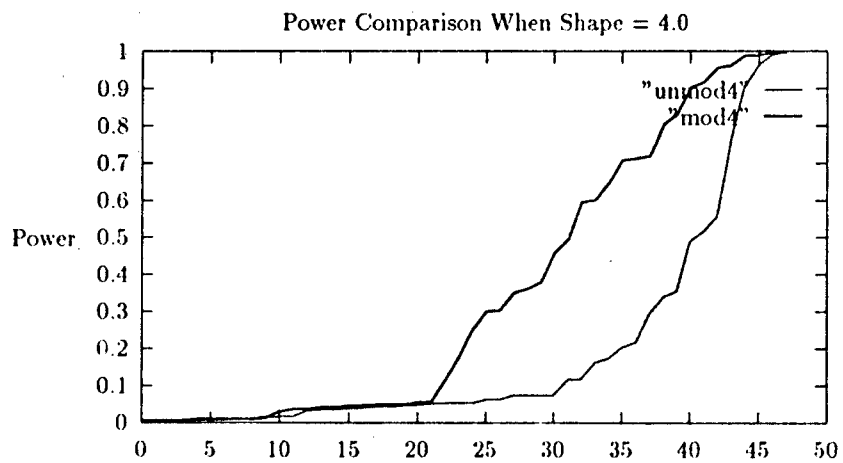


Figure 4.52. Graphical Comparison of the Previous and Modified Power Studies When Shape = 4.0 (Graph is prepared by taking the average of powers.)

V. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

Based on the results obtained in this thesis, the following conclusions are noted:

1. The first thing to say is that, the Anderson-Darling critical values for two-parameter Gamma are valid.
2. This type of studies are a kind of building new developments on the previous accumulated knowledge. Getting an improvement by way of a modification is the objective of the study. In this research, Anderson-Darling test was known as one of the most powerful test on the two parameter Gamma; and this thesis aimed to get an improvement on this test. A new modified test has been developed and considerable amount of increase has been observed on the power of the modified test. The comparison with the previously modified Anderson-Darling test over 104 cases appeared with 56% of the cases considerably better power, 21% of the cases the same and 23% of the cases slightly less power.
3. The power study in general showed that the power increase with the increase of sample size and significance level. Good power is achieved for alternate distributions such as Weibull, uniform, lognormal and especially beta.
4. The functional relationship between the critical values, sample size and significance level have been presented for each shape parameter. So that, goodness-of-fit tests can be done for sample sizes and significance levels other than those presented in this thesis.

5.2 *Recommendations*

First of all, for further reliability analysis or whenever needed, I strongly recommend this modified Anderson-Darling test as the known most powerful test for the case of two-parameter Gamma.

For further research other goodness-of-fit tests can be tried to develop a more efficient technique. Also, some experimental design studies can be investigated by using several different statistics.

Some more or different alternate distributions can be worked on.

It might be interesting to extend the Sinclair, Spurr and Ahmad's [44] A-D test study which gives greater weight to larger and smaller observations when the objective of the fitting process is to predict a quantile at the tails of the distribution.

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Appendix A. COMPUTER PROGRAMS

A.1 Finding The Critical Value Tables

```
C *****
C *   THIS PROGRAM CALCULATES THE ANDERSON DARLING CRITICAL VALUES   *
C *****
```

```
      PROGRAM AD
      COMMON/VALUE/P(100)
      COMMON/RAY/T(100)
      COMMON/MIN/IN
      COMMON/MIN1/XNCDF(50),DIFKS,I,IKS,IKS1
      COMMON/MIN2/DIFCVM,ICVM,ICV1
      COMMON/MIN3/DIFAD,IAD,IAD1
      COMMON/MANA/N,SS1,SS2,SS3,M,C1,T1,A1,MR
      DOUBLE PRECISION DSEED,T,C1,T1,A1,CSJ,ASJ,TSJ
      DOUBLE PRECISION CKS,CCVM,CAD
      DIMENSION FX(60),AA(5000),XX(5002),YY(5002)
      INTEGER REP,PP
      DSEED=1500.000
      MR=0
      NONE=0
      NZERO=0
      REP=5002
      NOS=REP-2
      NUM=REP-2
      YY(1)=0.
      YY(REP)=1.
      DO 405 L=2,REP-1
         YY(L)=((L-1)-.5)/NOS
405    CONTINUE

      DO 100 PP=25,40,5
      PRINT*,"PP",PP
      N=PP
      M=N
      IN=N
      CALL RNSET(DSEED)
      DO 99 KK=1,5000
      SS1=1
      SS2=0
```

```

SS3=1
C1=10
A1=1
T1=1

C *** GAMMA RANDOM NUMBER GENERATOR
CALL RNGAM(N,A1,P)
DO 719 IK=1,N
P(IK)=T1*P(IK)+C1
IF (KK.LT.3) THEN
PRINT*,"P",KK,P(IK)
ENDIF
719 CONTINUE

C *** SORT A REAL ARRAY BY ALGEBRAIC VALUE
CALL SVRGN(N,P,P)
DO 3 II=1,N
T(II)=P(II)
3 CONTINUE

C *** GAMMA MLE ESTIMATION
CALL GAMMLE(CSJ,TSJ,ASJ)
IF (KK.LT.5) THEN
PRINT*,"C T A",CSJ,TSJ,ASJ
ENDIF

C *** MINIMUM DISTANCE ESTIMATION
CALL MINDIS(ASJ,CSJ,TSJ,CKS,CCVM,CAD)
IF (KK.LT.5) THEN
PRINT*,"min",CKS,CCVM,CAD
ENDIF
C1=CAD
SS3=0

CALL GAMMLE(CSJ,TSJ,ASJ)
IF (KK.LT.5) THEN
PRINT*,"C T A son",CSJ,TSJ,ASJ
ENDIF
DO 333 L=1,N
W=(T(L)-CSJ)/TSJ
X1=ASJ
FX(L)=GAMDF(W,X1)

```

```

      IF(FX(L).EQ.0.) THEN
      FX(L)=FX(L)+.0001
      NZERO=NZERO+1
      END IF
      IF(FX(L).EQ.1.) THEN
      FX(L)=FX(L)-.0001
      NONE=NONE+1
      END IF
333  CONTINUE
      WAD=0.
      XN=N
      DO 500 I=1,N
      XI=I
      WAD=WAD+(2.*XI-1.)*(LOG(FX(I))+LOG(1.-FX(N+1-I)))
500  CONTINUE
      WAD=(-WAD/XN)-XN
      AA(KK)=WAD
99   CONTINUE
      CALL SVRGN(5000,AA,AA)
      DO 400 L=1,REP-2
      XX(L+1)=AA(L)
400  CONTINUE
      CALL ENDPT(XX,YY,REP,NUM)
      PRINT '(2X,A,I2)', 'FOR N= ',PP,"SHAPE",A1
      DO 410 J=80,95,5
      DO 420 II=1,REP
      I=REP+1-II
      IF(YY(I).LT.(J/100.0)) THEN
      SLOPE=(YY(I+1)-YY(I))/(XX(I+1)-XX(I))
      ZZ=-SLOPE*XX(I)+YY(I)
      PRINT*, "THE ",J," TH PERCENTILE IS",((J/100.0)-ZZ)/SLOPE
      GO TO 410
      END IF
420  CONTINUE
410  CONTINUE
      DO 430 AK=1,REP
      K=REP+1-AK
      IF(YY(K).LT..99) THEN
      GO TO 999
      END IF
430  CONTINUE
999  SLOPE=(YY(K+1)-YY(K))/(XX(K+1)-XX(K))

```



```

ZZ=-SLOPE*XX(K)+YY(K)
PRINT*,"THE    99  TH PERCENTILE IS",(.99-ZZ)/SLOPE
100 CONTINUE
END
SUBROUTINE ENDPT(XX,YY,REP,NUM)
INTEGER REP
DIMENSION XX(5002),YY(5002)
SLOPE=(YY(2)-YY(3))/(XX(2)-XX(3))
B=YY(2)-SLOPE*XX(2)
V1=-B/SLOPE
IF(V1.LT.0.)THEN
    V1=0.
ENDIF
XX(1)=V1
SLOPE=(YY(NUM)-YY(NUM+1))/(XX(NUM)-XX(NUM+1))
B1=YY(NUM)-SLOPE*XX(NUM)
V2=(1.-B1)/SLOPE
XX(REP)=V2
RETURN
END

```

```

C *****
C *   THIS SUBROUTINE IS FOR MAXIMUM LIKELIHOOD ESTIMATION   *
C *****

```

```

SUBROUTINE GAMMLE(CSJ,TSJ,ASJ)
COMMON/RAY/T(100)
COMMON/MANA/N,SS1,SS2,SS3,M,C1,T1,A1,MR
DOUBLE PRECISION T,C,THETA,ALPHA,DLT,DLC,CE,TH,EN,EM,ELNM
DOUBLE PRECISION EMR,EI,D2T,DT,D2A,DA,D2C,DC,ENS,GAM,GMA,GAMI,GMAI
DOUBLE PRECISION GMAI2,DEXP,DABS,DLOG,SL,SR,S1
DOUBLE PRECISION EL,CSJ,TSJ,ASJ,C1,T1,A1
DIMENSION C(1100),THETA(1100),ALPHA(1100)
DIMENSION DLT(50),DLC(50),CE(50),TH(50)
JI=20
JH=20
C(1)=C1
THETA(1)=T1
ALPHA(1)=A1
9   EN=N
    EM=M
86  ELNM=0.D0

```

```

      EMR=MR
      MRP=MR+1
87      NM=N-M+1
      DO 88 I=NM,N
      EI=I
88      ELNM=ELNM+DLOG(EI)
      IF(MR) 66,89,109
109     DO 110 I=1,MR
      EI=I
110     ELNM=ELNM-DLOG(EI)
89      DO 63 J=1,1100
      IF (J-1) 66,112,111
111     JJ=J-1
      IF (J-JI) 6,139,139
139     IF (J/JH-JJ/JH) 6,6,117
117     J2=J-2
      J3=J-3
      IF(SS1) 119,119,118
118     D2T=THETA(JJ)-2.D0*THETA(J2)+THETA(J3)
      DT=THETA(JJ)-THETA(J2)
      IF(D2T) 135,119,135
135     NT=DABS(DT/D2T)
      GO TO 120
119     NT=999999
120     IF(SS2) 122,122,121
121     D2A=ALPHA(JJ)-2.D0*ALPHA(J2)+ALPHA(J3)
      DA=ALPHA(JJ)-ALPHA(J2)
      IF(D2A) 136,122,136
136     NA=DABS(DA/D2A)
      GO TO 123
122     NA=999999
123     IF(SS3) 125,125,124
124     D2C=C(JJ)-2.D0*C(J2)+C(J3)
      DC=C(JJ)-C(J2)
      IF (C(JJ)+0.00005-T(1))140,125,125
140     IF (C(JJ)-0.00005)125,125,141
141     IF (D2C)137,125,137
137     NC=DABS(DC/D2C)
      GO TO 126
125     NC=999999
126     IF ((NT.LT.NC).AND.(NT.LT.NA))      THEN
      MIN=NT

```

```

        ELSEIF (NC.LT.NA) THEN
            MIN=NC
        ELSE
            MIN=NA
        ENDIF
        NS=2*MIN
        IF(NS)6,6,142
142    IF(NS-999999)138,6,6
138    ENS=NS
        THETA(J)=THETA(JJ)+(DT+.25D0*(ENS+1.D0)*D2T)*ENS
        IF (THETA(J).GT.1.D-4) THEN
            THETA(J)=THETA(J)
        ELSE
            THETA(J)=1.D-4
        ENDIF
130    ALPHA(J)=ALPHA(JJ)
        IF (SS3) 133,133,134
133    C(J)=C(JJ)
        GO TO 112
134    C(J)=C(JJ)+(DC+.25D0*(ENS+1.D0)*D2C)*ENS
        IF (C(J).GT.0.D-4) THEN
            C(J)=C(J)
        ELSE
            C(J)=0.D-4
        ENDIF
        IF (C(J).LT.T(1)) THEN
            C(J)=C(J)
        ELSE
            C(J)=T(1)
        ENDIF
        IF ((1.D0-EMR)*C(J)-T(1))112,6,6
6    THETA(J)=THETA(JJ)
        IF (SS1)13,13,7
7    S1=0.D0
        DO 8 I=MRP,M
8    S1=S1+T(I)-C(JJ)
        IF (N-M+MR)66,73,74
73    THETA(J)=S1/(EM*ALPHA(JJ))
        GO TO 13
74    GMA=GAM(ALPHA(JJ))
        KS=0
        DO 108 K=1,5000

```

```

KK=K-1
GMAI=GAMI((T(M)-C(JJ))/THETA(J),ALPHA(JJ))
GMAI2=GAMI((T(MRP)-C(JJ))/THETA(J),ALPHA(JJ))
DLT(K)=-EM*ALPHA(JJ)/THETA(J)+S1/THETA(J)**2+
1 (EN-EM)*(T(M)-C(JJ))**ALPHA(JJ)*DEXP((C(JJ)
1 -T(M))/THETA(J))/(THETA(J)**(ALPHA(JJ)+1.DO)*(GMA-GMAI)) +EMR*ALPHA(JJ)
2 /THETA(J)-EMR*(T(MRP)-C(JJ))**ALPHA(JJ)*DEXP((C(JJ)-T(MRP))
3 /THETA(J))/(THETA(J)**(ALPHA(JJ)+1.DO)*GMAI2)
TH(K)=THETA(J)
IF (DLT(K))101,13,102
101 KS=KS-1
IF (KS+K)105,103,105
102 KS=KS+1
IF (KS-K)105,104,105
103 THETA(J)=.5D0*TH(K)
GO TO 108
104 THETA(J)=1.5D0*TH(K)
GO TO 108
105 IF (DLT(K)*DLT(KK))107,13,106
106 AK=KK-1
GO TO 105
107 THETA(J)=TH(K)+DLT(K)*(TH(K)-TH(KK))/(DLT(KK)-DLT(K))
IF (DABS(THETA(J)-TH(K))-1.D-4)13,13,108
108 CONTINUE
13 ALPHA(J)=ALPHA(JJ)
44 C(J)=C(JJ)
85 IF (SS3)112,112,45
45 IF (1.D0-ALPHA(J))79,143,143
143 IF (SS1+SS2)57,57,79
79 IF (N-M)66,83,46
46 GMA=GAM(ALPHA(J))
83 KS=0
DO 56 K=1,50
KK=K-1
SR=0.D0
DO 69 I=MRP,M
69 SR=SR+1.D0/(T(I)-C(J))
IF (N-M+MR)66,80,81
80 DLC(K)=(1.D0-ALPHA(J))*SR+EM/THETA(J)
GO TO 82
81 GMAI=GAMI((T(M)-C(J))/THETA(J),ALPHA(J))
GMAI2=GAMI((T(MRP)-C(J))/THETA(J),ALPHA(J))

```

```

      DLC(K)=(1.D0-ALPHA(J))*SR+(EM-EMR)/THETA(J)+
1  (EN-EM)*(T(M)-C(J))**(ALPHA(J)-1.D0)*
4  DEXP(-(T(M)-C(J))/THETA(J))/(THETA(J)**ALPHA(J)*
2  (GMA-GMAI))-EMR*(T(MRP)-C(J))**(ALPHA(J)-1.D0)
3  *DEXP(-(T(MRP)-C(J))/THETA(J))/(THETA(J)**ALPHA(J)*GMAI2)
82  CE(K)=C(J)
51  IF (DLC(K))90,112,91
90  KS=KS-1
      IF (KS+K)54,52,54
91  KS=KS+1
      IF (KS-K)54,53,54
52  C(J)=.5D0*CE(K)
      GO TO 68
53  C(J)=CE(K)+.5D0*(T(1)-CE(K))
      GO TO 68
54  IF (DLC(K)*DLC(KK))67,112,55
55  KK=KK-1
      GO TO 54
67  C(J)=CE(K)+DLC(K)*(CE(K)-CE(KK))/(DLC(KK)-DLC(K))
68  IF (DABS(C(J)-CE(K))-1.D-4)112,112,56
56  CONTINUE
      GO TO 112
57  C(J)=T(1)
112  IF (MR)66,113,58
113  DO 115 I=1,M
      IF (C(J)+1.D-4-T(I))116,114,114
114  MR=MR+1
115  C(1)=T(1)
116  IF (MR)66,58,86
58  S1=0.D0
      SL=0.D0
      DO 92 I=MRP,M
      S1=S1+T(I)-C(J)
92  SL=SL+DLOG(T(I)-C(J))
      GMA=GAM(ALPHA(J))
      IF (N-M+MR)66,98,96
96  GMAI=GAMI((T(M)-C(J))/THETA(J),ALPHA(J))
      GMAI2=GAMI((T(MRP)-C(J))/THETA(J),ALPHA(J))
98  EL=ELNM-EM*DLOG(GMA)-EM*ALPHA(J)*DLOG(THETA(J))+(ALPHA(J)-1.D0)*SL
1-S1/THETA(J)
      IF (N-M+MR)66,100,99
99  EL=EL+(EN-EM)*(DLOG(GMA-GMAI)-DLOG(GMA))

```

```

1+EMR*ALPHA(J)*DLOG(THETA(J))+EMR*DLOG(GMAI2)
100  TSJ=THETA(J)
      ASJ=ALPHA(J)
      CSJ=C(J)
      IF (J-2)63,60,60
60   IF(DABS(C(J)-C(JJ))-1.D-4)61,61,63
61   IF(DABS(THETA(J)-THETA(JJ))-1.D-4)62,62,63
62   IF(DABS(ALPHA(J)-ALPHA(JJ))-1.D-4)4,4,63
63   CONTINUE
4    CONTINUE
66   RETURN
      END

      DOUBLE PRECISION FUNCTION GAM(Y)
      DOUBLE PRECISION G,Z,DLOG,DEXP,Y
      Z=Y
      G=0.D0
1    IF (Z-9.D0)2,2,3
2    G=G-DLOG(Z)
      Z=Z+1.D0
      GO TO 1
3    GAM=G+(Z-.5D0)*DLOG(Z)-Z+.5D0*DLOG(2.D0*3.141592653589793D0)+1.D0/(12.D0*Z
1 -1.D0/(360.D0*Z**3)+1.D0/(1260.D0*Z**5)-1.D0/(1680.D0*Z**
2 7)+1.D0/(1188.D0*Z**9)-691.D0/(360360.D0*Z**11)+1.D0/(156.D0*Z**13
3 )
      GAM=DEXP(GAM)
      RETURN
      END

C    FUNCTION DGAM
      DOUBLE PRECISION FUNCTION DGAM(Y)
      DOUBLE PRECISION DG,Z,Y,DLOG,GAM
      Z=Y
      DG=0.D0
1    IF (Z-9.D0)2,2,3
2    DG=DG-1.D0/Z
      Z=Z+1.D0
      GO TO 1
3    DGAM=DG+(Z-.5D0)/Z+DLOG(Z)-1.D0-1.D0/(12.D0*Z**2)+1.D0/(120.D0*Z**
1 4)-1.D0/(252.D0*Z**6)+1.D0/(240.D0*Z**8)-1.D0/(132.D0*Z**10)
2 +691.D0/(32760.D0*Z**12)-1.D0/(12.D0*Z**14)
      DGAM=DGAM*GAM(Y)

```

RETURN
END

```
C  FUNCTION DGAMI
    DOUBLE PRECISION FUNCTION DGAMI(W,Z)
    DOUBLE PRECISION U,V,W,Z,SU,ELL
    DIMENSION U(50),V(50)
    U(1)=W**Z*DLOG(W)/Z
    V(1)=W**Z/Z**2
    SU=U(1)-V(1)
    DO 1 L=2,50
    LL=L-1
    ELL=LL
    U(L)=(-U(LL)*W/ELL)*(Z+ELL-1.D0)/(Z+ELL)
    V(L)=-V(LL)*W*(Z+ELL-1.D0)**2/((Z+ELL)**2*ELL)
1   SU=SU+U(L)-V(L)
    DGAMI=SU
    RETURN
    END
```

```
C  FUNCTION GAMI
    DOUBLE PRECISION FUNCTION GAMI(W,Z)
    DOUBLE PRECISION U,W,Z,SU,ELL
    DIMENSION U(50)
    U(1)=W**Z/Z
    SU=U(1)
    DO 1 L=2,50
    LL=L-1
    ELL=LL
    U(L)=(-U(LL)/ELL)*W*(Z+ELL-1.D0)/(Z+ELL)
1   SU=SU+U(L)
    GAMI=SU
    RETURN
    END
```

```
C *****
C *   THIS SUBROUTINE IS FOR MINIMUM DISTANCE ESTIMATION   *
C *****
```

SUBROUTINE MINDIS(ASJ,CSJ,TSJ,CKS,CCVM,CAD)
DOUBLE PRECISION ASJ,CSJ,TSJ,AHAT,THAT,CHAT,CKS,CCVM,CAD,X2

```

      INTEGER ICKE, IKS1, ICV1, IAD1
      COMMON/MIN/IN
      COMMON/MIN1/XNCDF(50), DIFKS, I, IKS, IKS1
      COMMON/MIN2/DIFCVM, ICVM, ICV1
      COMMON/MIN3/DIFAD, IAD, IAD1
      COMMON/VALUE/P(100)
      AHAT=ASJ
      CHAT=CSJ
      THAT=TSJ
      N = IN
      DO 10 I=1,N
      XNCDF(I)=0.0
10      CONTINUE

      C *      COMPUTE MINIMUM DISTANCE ESTIMATES FOR LOCATION
      DIFKS = 9999999.99
      DIFCVM= 9999999.99
      DIFAD = 9999999.99
      IKS=0
      ICVM = 0
      IAD = 0
      X2 = P(1)-.0001
      CHAT = X2
      IKS1 = 0
      ICV1 = 0
      IAD1 = 0
      DO 200 I = 1,200
      X2 = X2 - .01
      DO 160 L=1,N
      ANORM=(P(L)-X2)/TSJ
      X1=ASJ
      XNCDF(L)=GAMDF(ANORM,X1)
      IF(XNCDF(L).EQ.0.) THEN
      XNCDF(L)=XNCDF(L)+.0001
      NZERO=NZERO+1
      END IF
      IF(XNCDF(L).EQ.1.) THEN
      XNCDF(L)=XNCDF(L)-.0001
      NONE=NONE+1
      END IF
160      CONTINUE
      IF (IKS1 .EQ. 1) GO TO 182

```



```

      CALL WKS(N)
182  CONTINUE
      IF (ICV1 .EQ. 1) GO TO 183
      CALL WCVN(N)
183  CONTINUE
      IF (IAD1 .EQ. 1) GO TO 198
      CALL WAD(N)
198  CONTINUE
      ICKE = IKS1+ICV1+IAD1
      IF (ICKE .EQ. 3) GO TO 201
200  CONTINUE
201  CONTINUE
      CKS = CHAT - 0.01*(IKS-1)
      CCVM = CHAT - 0.01*(ICVM-1)
      CAD = CHAT - 0.01*(IAD-1)
      RETURN
      END

C *** WEIGHTED K-S ***
      SUBROUTINE WKS(N)
      COMMON/MIN1/XNCDF(50),DIFKS,I,IKS,IKS1
      TOP = 0.0
      BOT = 0.0
      XN = N
      DO 10 L = 1,N
      RL = L
      IF(RL/XN-XNCDF(L) .GT. TOP) TOP = RL/XN - XNCDF(L)
      IF(XNCDF(L)-(RL-1)/XN .GT. BOT) BOT = XNCDF(L) - (RL-1)/XN
10   CONTINUE
      DIF = TOP
      IF(BOT .GT. DIF) DIF = BOT
      IF(DIF .LT. DIFKS) GO TO 20
      IKS1 = 1
      RETURN
20   IKS=I
      DIFKS = DIF
      RETURN
      END

C *** WEIGHTED C-V M ***
      SUBROUTINE WCVN(N)

```

```

COMMON/MIN1/XNCDF(50),DIFKS,I,IKS,IKS1
COMMON/MIN2/DIFCVM,ICVM,ICV1
XN = N
DFCVM = 0.0
DO 10 M = 1,N
XM = M
DFCVM = DFCVM + (XNCDF(M) - (2.*XM - 1.) / (2.*XN))**2
10  CONTINUE
DFCVM = DFCVM + 1./(12.*XN)
IF(DFCVM .LT. DIFCVM) GO TO 20
ICV1 = 1
RETURN
20  DIFCVM = DFCVM
ICVM = I
RETURN
END

C *** ANDERSON-DARLING ***
SUBROUTINE WAD(N)
COMMON/MIN1/XNCDF(50),DIFKS,I,IKS,IKS1
COMMON/MIN3/DIFAD,IAD,IAD1
DFAD = 0.0
DO 10 K = 1,N
RK = K
JK = N + 1 - K
IF(XNCDF(JK) .GE. 1.0) XNCDF(JK) = .999999999
DFAD = DFAD + (2.*RK-1.)*(LOG(XNCDF(K))+LOG(1.-XNCDF(JK)))
10  CONTINUE
DFAD = ABS(-DFAD/N-N)
IF(DFAD .LT. DIFAD) GO TO 20
IAD1 = 1
RETURN
20  DIFAD = DFAD
IAD = I
RETURN
END

```

Appendix B. COMPUTER PROGRAMS

B.1 Power Study

C THIS PROGRAM IS A SAMPLE FOR POWER STUDY (SHAPE=1)

```
PROGRAM AD
COMMON/VALUE/P(100)
COMMON/RAY/T(100)
COMMON/MIN/IN
COMMON/MIN1/XNCDF(60),DIFKS,I,IKS,IKS1
COMMON/MIN2/DIFCVM,ICVM,ICV1
COMMON/MIN3/DIFAD,IAD,IAD1
COMMON/MANA/N,SS1,SS2,SS3,M,C1,T1,A1,MR
DOUBLE PRECISION DSEED,T,C1,T1,A1,CSJ,ASJ,TSJ
DOUBLE PRECISION CKS,CCVM,CAD
REAL ALPHA20,ALPHA15,ALPHA10,ALPHA05,ALPHA01,VAR1,MU
DIMENSION FX(60),AA(5000),XX(5002),YY(5002)
INTEGER REP,PP
DSEED=1500.000
MR=0
NONE=0
NZERO=0
ALPHA20=0.
ALPHA15=0.
ALPHA10=0.
ALPHA05=0.
ALPHA01=0.
REP=5002
NOS=REP-2
NUM=REP-2
YY(1)=0.
YY(REP)=1.
DO 405 L=2,REP-1
    YY(L)=((L-1)-.5)/NOS
405 CONTINUE
CALL RNSET(DSEED)
DO 300 SS=1,10
DO 100 PP=5,30,5
    PRINT*,"PP",PP
N=PP
M=N
```

```

IN=N
DC 99 KK=1,5000

C    CALL GGAMR(DSEED,A1,N,G,P)
      SS1=1
      SS2=0
      SS3=1
      C1=10
      A1=1.
      T1=1
      IF (SS.EQ.1) GOTO 401
      IF (SS.EQ.2) GOTO 402
      IF (SS.EQ.3) GOTO 403
      IF (SS.EQ.4) GOTO 404
      IF (SS.EQ.5) GOTO 406
      IF (SS.EQ.6) GOTO 407
      IF (SS.EQ.7) GOTO 408
      IF (SS.EQ.8) GOTO 409
      IF (SS.EQ.9) GOTO 410
      IF (SS.EQ.10) GOTO 411

401   CALL RNGAM(N,A1,P)
      DO 719 IK=1,N
      P(IK)=T1*P(IK)+C1
      IF (KK.LT.3) THEN
      PRINT*,"P",KK,P(IK)
      ENDIF

719   CONTINUE
      GOTO 777

402   A2=2.5
      CALL RNGAM(N,A2,P)
      DO 718 IK=1,N
      P(IK)=T1*P(IK)+C1
      IF (KK.LT.3) THEN
      PRINT*,"P",KK,P(IK)
      ENDIF

718   CONTINUE
      GOTO 777

403   A3=4.
      CALL RNGAM(N,A3,P)

```

```

DO 717 IK=1,N
P(IK)=T1*P(IK)+C1
IF (KK.LT.3) THEN
PRINT*,"P",KK,P(IK)
ENDIF

717  CONTINUE
GOTO 777

404  A2=2.
CALL RNWIB(N,A2,P)
DO 716 IK=1,N
P(IK)=T1*P(IK)+C1
IF (KK.LT.3) THEN
PRINT*,"P",KK,P(IK)
ENDIF

716  CONTINUE
GOTO 777

406  A3=3.
CALL RNWIB(N,A3,P)
DO 715 IK=1,N
P(IK)=T1*P(IK)+C1
IF (KK.LT.3) THEN
PRINT*,"P",KK,P(IK)
ENDIF

715  CONTINUE
GOTO 777

407  CALL RNUN(N,P)
DO 709 IK=1,N
P(IK)=5*P(IK)+C1
IF (KK.LT.3) THEN
PRINT*,"P",KK,P(IK)
ENDIF

709  CONTINUE
GOTO 777

408  XM=0.
S2=1.
CALL RNLNL(N,XM,S2,P)
DO 713 IK=1,N

```

```

P(IK)=P(IK)+C1
IF (KK.LT.3) THEN
PRINT*,"P",KK,P(IK)
ENDIF

713  CONTINUE
      GOTO 777

409  XM=0.
      S2=2.
      CALL RNLNL(N,XM,S2,P)
      DO 712 IK=1,N
        P(IK)=P(IK)+C1
        IF (KK.LT.3) THEN
          PRINT*,"P",KK,P(IK)
        ENDIF

712  CONTINUE
      GOTO 777

410  PIN=1.0
      QIN=1.0
      CALL RNBET(N,PIN,QIN,P)
      DO 711 IK=1,N
        P(IK)=P(IK)+C1
        IF (KK.LT.3) THEN
          PRINT*,"P",KK,P(IK)
        ENDIF

711  CONTINUE
      GOTO 777

411  PIN=2.0
      QIN=2.0
      CALL RNBET(N,PIN,QIN,P)
      DO 710 IK=1,N
        P(IK)=P(IK)+C1
        IF (KK.LT.3) THEN
          PRINT*,"P",KK,P(IK)
        ENDIF

710  CONTINUE

C    CALL VSRTA(P,N)
777  CALL SVRGN(N,P,P)
      DO 3 II=1,N

```

```

3      T(II)=P(II)
      CONTINUE
      CALL GAMMLE(CSJ,TSJ,ASJ)
      IF (KK.LT.5) THEN
        PRINT*,"C T A",CSJ,TSJ,ASJ
      ENDIF
      CALL MINDIS(ASJ,CSJ,TSJ,CKS,CCVM,CAD)
      IF (KK.LT.5) THEN
        PRINT*,"min",CKS,CCVM,CAD
      ENDIF
      C1=CAD
      SS3=0
      CALL GAMMLE(CSJ,TSJ,ASJ)
      IF (KK.LT.5) THEN
        PRINT*,"C T A son",CSJ,TSJ,ASJ
      ENDIF
      DO 333 L=1,N
        W=(T(L)-CSJ)/TSJ
        X1=ASJ

        FX(L)=GAMDF(W,X1)
        IF(FX(L).EQ.0.) THEN
          FX(L)=FX(L)+.0001
          NZERO=NZERO+1
        END IF
        IF(FX(L).EQ.1.) THEN
          FX(L)=FX(L)-.0001
          NONE=NONE+1
        END IF

333    CONTINUE
        WAD=0.
        XN=N
        DO 500 I=1,N
          XI=I
          WAD=WAD+(2.*XI-1.)*(LOG(FX(I))+LOG(1.-FX(N+1-I)))
500    CONTINUE
          WAD=(-WAD/XN)-XN
          AA(KK)=WAD
99     CONTINUE

      CALL SVRGN(5000,AA,AA)

```

```

DO 400 L=1,REP-2
XX(L+1)=AA(L)
400 CONTINUE
IF (PP.EQ.5) GOTO 501
IF (PP.EQ.10) GOTO 502
IF (PP.EQ.15) GOTO 503
IF (PP.EQ.20) GOTO 504
IF (PP.EQ.25) GOTO 506
IF (PP.EQ.30) GOTO 507
IF (PP.EQ.35) GOTO 508
IF (PP.EQ.40) GOTO 509
501 DO 78 Z=1,REP

C * COMPARE WITH CRITICAL VALUES

IF (AA(Z).GE. 0.674343) ALPHA20=ALPHA20+1
IF (AA(Z).GE. 0.726905) ALPHA15=ALPHA15+1
IF (AA(Z).GE. 0.801290) ALPHA10=ALPHA10+1
IF (AA(Z).GE. 0.904187) ALPHA05=ALPHA05+1
IF (AA(Z).GE. 1.152740) ALPHA01=ALPHA01+1
78 CONTINUE
GOTO 365
502 DO 79 Z=1,REP
IF (AA(Z).GE. 0.799794) ALPHA20=ALPHA20+1
IF (AA(Z).GE. 0.894774) ALPHA15=ALPHA15+1
IF (AA(Z).GE. 1.016620) ALPHA10=ALPHA10+1
IF (AA(Z).GE. 1.211040) ALPHA05=ALPHA05+1
IF (AA(Z).GE. 1.603610) ALPHA01=ALPHA01+1
79 CONTINUE
GOTO 365
503 DO 85 Z=1,REP
IF (AA(Z).GE. 1.06126) ALPHA20=ALPHA20+1
IF (AA(Z).GE. 1.17946) ALPHA15=ALPHA15+1
IF (AA(Z).GE. 1.35270) ALPHA10=ALPHA10+1
IF (AA(Z).GE. 1.61627) ALPHA05=ALPHA05+1
IF (AA(Z).GE. 2.07047) ALPHA01=ALPHA01+1
85 CONTINUE
GOTO 365
504 DO 80 Z=1,REP
IF (AA(Z).GE. 1.23485) ALPHA20=ALPHA20+1
IF (AA(Z).GE. 1.35969) ALPHA15=ALPHA15+1
IF (AA(Z).GE. 1.53478) ALPHA10=ALPHA10+1

```



```

      IF (AA(Z).GE. 1.80224) ALPHA05=ALPHA05+1
      IF (AA(Z).GE. 2.31085) ALPHA01=ALPHA01+1
80    CONTINUE
      GOTO 365
506    DO 81 Z=1,REP
      IF (AA(Z).GE. 1.34786) ALPHA20=ALPHA20+1
      IF (AA(Z).GE. 1.48064) ALPHA15=ALPHA15+1
      IF (AA(Z).GE. 1.66627) ALPHA10=ALPHA10+1
      IF (AA(Z).GE. 1.97667) ALPHA05=ALPHA05+1
      IF (AA(Z).GE. 2.56099) ALPHA01=ALPHA01+1
81    CONTINUE
      GOTO 365
507    DO 82 Z=1,REP
      IF (AA(Z).GE. 1.49442) ALPHA20=ALPHA20+1
      IF (AA(Z).GE. 1.65071) ALPHA15=ALPHA15+1
      IF (AA(Z).GE. 1.84893) ALPHA10=ALPHA10+1
      IF (AA(Z).GE. 2.17056) ALPHA05=ALPHA05+1
      IF (AA(Z).GE. 2.90480) ALPHA01=ALPHA01+1
82    CONTINUE
      GOTO 365
508    DO 83 Z=1,REP
      IF (AA(Z).GE. 1.61408) ALPHA20=ALPHA20+1
      IF (AA(Z).GE. 1.79560) ALPHA15=ALPHA15+1
      IF (AA(Z).GE. 2.00810) ALPHA10=ALPHA10+1
      IF (AA(Z).GE. 2.36925) ALPHA05=ALPHA05+1
      IF (AA(Z).GE. 2.94969) ALPHA01=ALPHA01+1
83    CONTINUE
      GOTO 365
509    DO 84 Z=1,REP
      IF (AA(Z).GE. 1.79139) ALPHA20=ALPHA20+1
      IF (AA(Z).GE. 1.97671) ALPHA15=ALPHA15+1
      IF (AA(Z).GE. 2.19041) ALPHA10=ALPHA10+1
      IF (AA(Z).GE. 2.54939) ALPHA05=ALPHA05+1
      IF (AA(Z).GE. 3.35005) ALPHA01=ALPHA01+1
84    CONTINUE
365    PRINT*,"SS = ",SS," PP ",PP
      PRINT*,"ALPHA20",ALPHA20/REP
      PRINT*,"ALPHA15",ALPHA15/REP
      PRINT*,"ALPHA10",ALPHA10/REP
      PRINT*,"ALPHA05",ALPHA05/REP
      PRINT*,"ALPHA01",ALPHA01/REP
      ALPHA20=0.

```

```

        ALPHA15=0.
        ALPHA10=0.
        ALPHA05=0.
        ALPHA01=0.
100     CONTINUE
300     CONTINUE
        END

```

```

SUBROUTINE ENDPT(XX,YY,REP,NUM)
INTEGER REP
DIMENSION XX(5002),YY(5002)
SLOPE=(YY(2)-YY(3))/(XX(2)-XX(3))
B=YY(2)-SLOPE*XX(2)
V1=-B/SLOPE
IF(V1.LT.0.)THEN
V1=0.
ENDIF

```

```

        XX(1)=V1
        SLOPE=(YY(NUM)-YY(NUM+1))/(XX(NUM)-XX(NUM+1))
        B1=YY(NUM)-SLOPE*XX(NUM)
        V2=(1.-B1)/SLOPE
        XX(REP)=V2
        RETURN
        END

```

```

SUBROUTINE GAMMLE(CSJ,TSJ,ASJ)
COMMON/RAY/T(100)
COMMON/MANA/N,SS1,SS2,SS3,M,C1,T1,A1,MR
DOUBLE PRECISION T,C,THETA,ALPHA,DLT,DLC,CE,TH,EN,EM,ELNM
DOUBLE PRECISION EMR,EI,D2T,DT,D2A,DA,D2C,DC,ENS,GAM,GMA,GAMI,GMAI
DOUBLE PRECISION GMAI2,DEXP,DABS,DLOG,SL,SR,S1
DOUBLE PRECISION EL,CSJ,TSJ,ASJ,C1,T1,A1
DIMENSION C(1100),THETA(1100),ALPHA(1100)
DIMENSION DLT(50),DLC(50),CE(50),TH(50)
JI=20
JH=20
C(1)=C1
THETA(1)=T1
ALPHA(1)=A1
9      EN=N
        EM=M

```

```

86      ELNM=0.D0
        EMR=MR
        MRP=MR+1
87      NM=N-M+1
        DO 88 I=NM,N
        EI=I
88      ELNM=ELNM+DLOG(EI)
        IF(MR) 66,89,109
109     DO 110 I=1,MR
        EI=I
110     ELNM=ELNM-DLOG(EI)
89      DO 63 J=1,1100
        IF (J-1) 66,112,111
111     JJ=J-1
        IF (J-JI) 6,139,139
139     IF (J/JH-JJ/JH) 6,6,117
117     J2=J-2
        J3=J-3
        IF(SS1) 119,119,118
118     D2T=THETA(JJ)-2.D0*THETA(J2)+THETA(J3)
        DT=THETA(JJ)-THETA(J2)
        IF(D2T) 135,119,135
135     NT=DABS(DT/D2T)
        GO TO 120
119     NT=9999999
120     IF(SS2) 122,122,121
121     D2A=ALPHA(JJ)-2.D0*ALPHA(J2)+ALPHA(J3)
        DA=ALPHA(JJ)-ALPHA(J2)
        IF(D2A) 136,122,136
136     NA=DABS(DA/D2A)
        GO TO 123
122     NA=9999999
123     IF(SS3) 125,125,124
124     D2C=C(JJ)-2.D0*C(J2)+C(J3)
        DC=C(JJ)-C(J2)
        IF (C(JJ)+0.00005-T(1))140,125,125
140     IF (C(JJ)-0.00005)125,125,141
141     IF (D2C)137,125,137
137     NC=DABS(DC/D2C)
        GO TO 126
125     NC=9999999
126     IF ((NT.LT.NC).AND.(NT.LT.NA))      THEN

```

```

        MIN=NT
        ELSEIF (NC.LT.NA) THEN
            MIN=NC
        ELSE
            MIN=NA
        ENDIF

        NS=2*MIN
        IF(NS)6,6,142
142    IF(NS-999999)138,6,6
138    ENS=NS
        THETA(J)=THETA(JJ)+(DT+.25D0*(ENS+1.D0)*D2T)*ENS
        IF (THETA(J).GT.1.D-4) THEN
            THETA(J)=THETA(J)
        ELSE
            THETA(J)=1.D-4
        ENDIF
130    ALPHA(J)=ALPHA(JJ)
        IF (SS3) 133,133,134
133    C(J)=C(JJ)
        GO TO 112
134    C(J)=C(JJ)+(DC+.25D0*(ENS+1.D0)*D2C)*ENS
        IF (C(J).GT.0.D-4) THEN
            C(J)=C(J)
        ELSE
            C(J)=0.D-4
        ENDIF
        IF (C(J).LT.T(1)) THEN
            C(J)=C(J)
        ELSE
            C(J)=T(1)
        ENDIF
        IF ((1.D0-EMR)*C(J)-T(1))112,6,6
6        THETA(J)=THETA(JJ)
        IF (SS1)13,13,7
7        S1=0.D0
        DO 8 I=MRP,M
8        S1=S1+T(I)-C(JJ)
        IF (N-M+MR)66,73,74
73    THETA(J)=S1/(EM*ALPHA(JJ))
        GO TO 13
74    GMA=GAM(ALPHA(JJ))

```

```

      KS=0
      DO 108 K=1,5000
      KK=K-1
      GMAI=GAMI((T(M)-C(JJ))/THETA(J),ALPHA(JJ))
      GMAI2=GAMI((T(MRP)-C(JJ))/THETA(J),ALPHA(JJ))
      DLT(K)=-EM*ALPHA(JJ)/THETA(J)+S1/THETA(J)**2+
1 (EN-EM)*(T(M)-C(JJ))**ALPHA(JJ)*DEXP((C(JJ)
1 -T(M))/THETA(J))/(THETA(J)**(ALPHA(JJ)+1.DO)*(GMA-GMAI)) +EMR*ALPHA(JJ)
2 /THETA(J)-EMR*(T(MRP)-C(JJ))**ALPHA(JJ)*DEXP((C(JJ)-T(MRP))
3 /THETA(J))/(THETA(J)**(ALPHA(JJ)+1.DO)*GMAI2)
      TH(K)=THETA(J)
      IF (DLT(K))101,13,102
101  KS=KS-1
      IF (KS+K)105,103,105
102  KS=KS+1
      IF (KS-K)105,104,105
103  THETA(J)=.5D0*TH(K)
      GO TO 108
104  THETA(J)=1.5D0*TH(K)
      GO TO 108
105  IF (DLT(K)*DLT(KK))107,13,106
106  KK=KK-1
      GO TO 105
107  THETA(J)=TH(K)+DLT(K)*(TH(K)-TH(KK))/(DLT(KK)-DLT(K))
      IF (DABS(THETA(J)-TH(K))-1.D-4)13,13,108
108  CONTINUE
13   ALPHA(J)=ALPHA(JJ)
44   C(J)=C(JJ)
85   IF (SS3)112,112,45
45   IF (1.D0-ALPHA(J))79,143,143
143  IF (SS1+SS2)57,57,79
79   IF (N-M)66,83,46
46   GMA=GAM(ALPHA(J))
83   KS=0
      DO 56 K=1,50
      KK=K-1
      SR=0.D0
      DO 69 I=MRP,M
69   SR=SR+1.D0/(T(I)-C(J))
      IF (N-M+MR)66,80,81
80   DLC(K)=(1.D0-ALPHA(J))*SR+EM/THETA(J)
      GO TO 82

```

```

81   GMAI=GAMI((T(M)-C(J))/THETA(J),ALPHA(J))
      GMAI2=GAMI((T(MRP)-C(J))/THETA(J),ALPHA(J))
      DLC(K)=(1.D0-ALPHA(J))*SR+(EM-EMR)/THETA(J)+
1   (EN-EM)*(T(M)-C(J))**(ALPHA(J)-1.D0)*
4   DEXP(-(T(M)-C(J))/THETA(J))/(THETA(J)**ALPHA(J)*
2   (GMA-GMAI))-EMR*(T(MRP)-C(J))**(ALPHA(J)-1.D0)
3   *DEXP(-(T(MRP)-C(J))/THETA(J))/(THETA(J)**ALPHA(J)*GMAI2)
82   CE(K)=C(J)
51   IF (DLC(K))90,112,91
90   KS=KS-1
      IF (KS+K)54,52,54
91   KS=KS+1
      IF (KS-K)54,53,54
52   C(J)=.5D0*CE(K)
      GO TO 68
53   C(J)=CE(K)+.5D0*(T(1)-CE(K))
      GO TO 68
54   IF (DLC(K)*DLC(KK))67,112,55
55   KK=KK-1
      GO TO 54
67   C(J)=CE(K)+DLC(K)*(CE(K)-CE(KK))/(DLC(KK)-DLC(K))
68   IF (DABS(C(J)-CE(K))-1.D-4)112,112,56
56   CONTINUE
      GO TO 112
57   C(J)=T(1)
112  IF (MR)66,113,58
113  DO 115 I=1,M
      IF (C(J)+1.D-4-T(I))116,114,114
114  MR=MR+1
115  C(1)=T(1)
116  IF (MR)66,58,86
58   S1=0.D0
      SL=0.D0
      DO 92 I=MRP,M
        S1=S1+T(I)-C(J)
92   SL=SL+DLOG(T(I)-C(J))
      GMA=GAM(ALPHA(J))
      IF (N-M+MR)66,98,96
96   GMAI=GAMI((T(M)-C(J))/THETA(J),ALPHA(J))
      GMAI2=GAMI((T(MRP)-C(J))/THETA(J),ALPHA(J))
98   EL=ELNM-EM*DLOG(GMA)-EM*ALPHA(J)*DLOG(THETA(J))+(ALPHA(J)-1.D0)*SL
      1-S1/THETA(J)

```

```

        IF (N-M+MR) 66,100,99
99      EL=EL+(EN-EM)*(DLOG(GMA-GMAI)-DLOG(GMA))
        1+EMR*ALPHA(J)*DLOG(THETA(J))+EMR*DLOG(GMAI2)
100     TSJ=THETA(J)
        ASJ=ALPHA(J)
        CSJ=C(J)
        IF (J-2) 63,60,60
60      IF(DABS(C(J)-C(JJ))-1.D-4) 61,61,63
61      IF(DABS(THETA(J)-THETA(JJ))-1.D-4) 62,62,63
62      IF(DABS(ALPHA(J)-ALPHA(JJ))-1.D-4) 4,4,63
63      CONTINUE
4       CONTINUE
66      RETURN
        END

```

```

        DOUBLE PRECISION FUNCTION GAM(Y)
        DOUBLE PRECISION G,Z,DLOG,DEXP,Y
        Z=Y
        G=0.D0
1       IF (Z-9.D0) 2,2,3
2       G=G-DLOG(Z)
        Z=Z+1.D0
        GO TO 1
3       GAM=G+(Z-.5D0)*DLOG(Z)-Z+.5D0*DLOG(2.D0*3.141592653589793D0)+1.D0/(12.D0
1 -1.D0/(360.D0*Z**3)+1.D0/(1260.D0*Z**5)-1.D0/(1680.D0*Z**
2 7)+1.D0/(1188.D0*Z**9)-691.D0/(360360.D0*Z**11)+1.D0/(156.D0*Z**13
3 )
        GAM=DEXP(GAM)
        RETURN
        END

```

```

C      FUNCTION DGAM
        DOUBLE PRECISION FUNCTION DGAM(Y)
        DOUBLE PRECISION DG,Z,Y,DLOG,GAM
        Z=Y
        DG=0.D0
1       IF (Z-9.D0) 2,2,3
2       DG=DG-1.D0/Z
        Z=Z+1.D0
        GO TO 1
3       DGAM=DG+(Z-.5D0)/Z+DLOG(Z)-1.D0-1.D0/(12.D0*Z**2)+1.D0/(120.D0*Z**

```

```

1      4)-1.D0/(252.D0*Z**6)+1.D0/(240.D0*Z**8)-1.D0/(132.D0*Z**10)
2      +691.D0/(32760.D0*Z**12)-1.D0/(12.D0*Z**14)

```

```

DGAM=DGAM*GAM(Y)

```

```

RETURN

```

```

END

```

```

C      FUNCTION DGAMI

```

```

      DOUBLE PRECISION FUNCTION DGAMI(W,Z)

```

```

      DOUBLE PRECISION U,V,W,Z,SU,ELL

```

```

      DIMENSION U(50),V(50)

```

```

      U(1)=W**Z*DLOG(W)/Z

```

```

      V(1)=W**Z/Z**2

```

```

      SU=U(1)-V(1)

```

```

      DO 1 L=2,50

```

```

      LL=L-1

```

```

      ELL=LL

```

```

      U(L)=(-U(LL)*W/ELL)*(Z+ELL-1.D0)/(Z+ELL)

```

```

      V(L)=-V(LL)*W*(Z+ELL-1.D0)**2/((Z+ELL)**2*ELL)

```

```

1      SU=SU+U(L)-V(L)

```

```

      DGAMI=SU

```

```

      RETURN

```

```

      END

```

```

C      FUNCTION GAMI

```

```

      DOUBLE PRECISION FUNCTION GAMI(W,Z)

```

```

      DOUBLE PRECISION U,W,Z,SU,ELL

```

```

      DIMENSION U(50)

```

```

      U(1)=W**Z/Z

```

```

      SU=U(1)

```

```

      DO 1 L=2,50

```

```

      LL=L-1

```

```

      ELL=LL

```

```

      U(L)=(-U(LL)/ELL)*W*(Z+ELL-1.D0)/(Z+ELL)

```

```

1      SU=SU+U(L)

```

```

      GAMI=SU

```

```

      RETURN

```

```

      END

```

```

SUBROUTINE MINDIS(ASJ,CSJ,TSJ,CKS,CCVM,CAD)

```

```

DOUBLE PRECISION ASJ,CSJ,TSJ,AHAT,THAT,CHAT,CKS,CCVM,CAD,X2

```



```

      INTEGER ICKE, IKS1, ICV1, IAD1
      COMMON/MIN/IN
      COMMON/MIN1/XNCDF(60), DIFKS, I, IKS, IKS1
      COMMON/MIN2/DIFCVM, ICVM, ICV1
      COMMON/MIN3/DIFAD, IAD, IAD1
      COMMON/VALUE/P(100)
      AHAT=ASJ
      CHAT=CSJ
      THAT=TSJ
      N = IN
      DO 10 I=1,N
      XNCDF(I)=0.0
10      CONTINUE

C *   COMPUTE MINIMUM DISTANCE ESTIMATES FOR LOCATION

      DIFKS = 9999999.99
      DIFCVM= 9999999.99
      DIFAD = 9999999.99
      IKS=0
      ICVM = 0
      IAD = 0
      X2 = P(1)-.0001
      CHAT = X2
      IKS1 = 0
      ICV1 = 0
      IAD1 = 0
      DO 200 I = 1,200
      X2 = X2 - .01
      DO 160 L=1,N
      ANORM=(P(L)-X2)/TSJ
      X1=ASJ
      XNCDF(L)=GAMDF(ANORM,X1)
      IF(XNCDF(L).EQ.0.) THEN
      XNCDF(L)=XNCDF(L)+.0001
      NZERO=NZERO+1
      END IF
      IF(XNCDF(L).EQ.1.) THEN
      XNCDF(L)=XNCDF(L)-.0001
      NONE=NONE+1
      END IF

```

```

160  CONTINUE
      IF (IKS1 .EQ. 1) GO TO 182
      CALL WKS(N)
182  CONTINUE
      IF (ICV1 .EQ. 1) GO TO 183
      CALL WCVN(N)
183  CONTINUE
      IF (IAD1 .EQ. 1) GO TO 198
      CALL WAD(N)
198  CONTINUE
      ICKE = IKS1+ICV1+IAD1
      IF (ICKE .EQ. 3) GO TO 201
200  CONTINUE
201  CONTINUE
      CKS = CHAT - 0.01*(IKS-1)
      CCVM = CHAT - 0.01*(ICVM-1)
      CAD = CHAT - 0.01*(IAD-1)
      RETURN
      END

```

C *** WEIGHTED K-S ***

```

      SUBROUTINE WKS(N)
      COMMON/MIN1/XNCDF(60),DIFKS,I,IKS,IKS1
      TOP = 0.0
      BOT = 0.0
      XN = N
      DO 10 L = 1,N
      RL = L
      IF(RL/XN-XNCDF(L) .GT. TOP) TOP = RL/XN - XNCDF(L)
      IF(XNCDF(L)-(RL-1)/XN .GT. BOT) BOT = XNCDF(L) - (RL-1)/XN
10   CONTINUE
      DIF = TOP
      IF(BOT .GT. DIF) DIF = BOT
      IF(DIF .LT. DIFKS) GO TO 20
      IKS1 = 1
      RETURN
20   IKS=I
      DIFKS = DIF
      RETURN
      END

```

C *** WEIGHTED C-V M ***

SUBROUTINE WCVN(N)

COMMON/MIN1/XNCDF(60),DIFKS,I,IKS,IKS1

COMMON/MIN2/DIFCVM,ICVM,ICV1

XN = N

DFCVM = 0.0

DO 10 M = 1,N

XM = M

DFCVM = DFCVM + (XNCDF(M) - (2.*XM - 1.) / (2.*XN))**2

10 CONTINUE

DFCVM = DFCVM + 1./(12.*XN)

IF(DFCVM .LT. DIFCVM) GO TO 20

ICV1 = 1

RETURN

20 DIFCVM = DFCVM

ICVM = I

RETURN

END

C *** ANDERSON-DARLING ***

SUBROUTINE WAD(N)

COMMON/MIN1/XNCDF(60),DIFKS,I,IKS,IKS1

COMMON/MIN3/DIFAD,IAD,IAD1

DFAD = 0.0

DO 10 K = 1,N

RK = K

JK = N + 1 - K

IF(XNCDF(JK) .GE. 1.0) XNCDF(JK) = .999999999

DFAD = DFAD + (2.*RK-1.)*(LOG(XNCDF(K))+LOG(1.-XNCDF(JK)))

10 CONTINUE

DFAD = ABS(-DFAD/N-N)

IF(DFAD .LT. DIFAD) GO TO 20

IAD1 = 1

RETURN

20 DIFAD = DFAD

IAD = I

RETURN

END

Vita

1Lt Tamer Özmen was born on 15 February 1965 in Kütahya TURKEY. He graduated from the Kuleli Military High School in 1983 and entered the Turkish Air Force Academy. He studied aeronautical engineering. 1Lt Özmen graduated from the Academy as a Second Lieutenant on 30 August 1987.

After graduating from the Air Defence School in 1989, he was assigned to İzmir radar site as an intercept controller.

1Lt Özmen worked for two years as an IC and was selected for the Postgraduate Education Program. He entered the School of Engineering, Air Force Institute of Technology, WPAFB, OH in 1991.

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Master's Thesis

**A MODIFIED ANDERSON-DARLING GOODNESS-OF-FIT TEST
FOR THE GAMMA DISTRIBUTION WITH UNKNOWN SCALE
AND LOCATION PARAMETERS**

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11. SUMMARY STATEMENT

12. DISTRIBUTION STATEMENT

13. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited

13. ABSTRACT

A new modified Anderson-Darling goodness-of-fit test is introduced for the three-parameter Gamma distribution when the location parameter is found by minimum distance estimation and scale parameter by maximum likelihood estimation. Monte Carlo simulation studies were performed to calculate the critical values for A-D test when A-D statistic is minimized. These critical values are then used for testing whether a set of observations follows a Gamma distribution when the scale and location parameters are unspecified and are estimated from the sample. Functional relationship between the critical values of A-D is also examined for each shape parameter by the variables, sample size (n) and significance level (α). The power study is performed with the hypothesized Gamma against alternate distributions. Comparison with the previous study which uses MLEs for location and scale showed that the modified test is better in most cases.

14. SUBJECT TERMS

Gamma Distribution; Goodness-of-Fit; Monte Carlo Simulation; Anderson-Darling; Minimum Distance; Maximum Likelihood; Parameter Estimation

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17. SECURITY CLASSIFICATION

Unclassified

18. SECURITY CLASSIFICATION

Unclassified

19. SECURITY CLASSIFICATION

Unclassified

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